AP Calculus AB Summer Assignment

Welcome to AP Calculus AB! I am very excited to teach you next year! Calculus is a fun and challenging course. Most people who have taken Calculus in the past remember it as being their favorite type of math. If I were to guess the reason for this, it is because Calculus ties together all the math you have ever learned since your parents taught you to count to the last thing you learned last semester. Calculus where the narrative of math comes together. Most students who take Calculus do not find the concepts of Calculus to be particularly hard. When a student struggles in Calculus it is because they have forgotten something that was taught to them perhaps many years ago. In Calculus you need to be able to recall anything you have ever learned in math at the drop of a hat. Unfortunately, due to the schedule of an AP course, we will not have time to review these past concepts in class. Every student will hit a point in Calculus when they need to go back and review something that they are a little rusty on.

This Summer Assignment that you have before you is meant to do several things, one of which is to give you an opportunity to go back and review concepts you are a little rusty on. Please try to cultivate the skill of going back and practicing older, because you will have to do this during the course of next year. Another reason for this summer assignment is to prepare you for our first test of the year. **Our first test will be given on the first block day of the school year.** (Please see the JSerra Academic Calendar for this date, and know it is subject to change throughout the summer break.) **This summer packet will act as a study guide for the first test.** Please know that you may or may not be allowed a calculator on this test, so please make sure you can do everything in this packet both with and without a calculator. A third reason for this summer assignment is to ensure you are serious about taking AP Calculus AB. An AP class is a lot of work in the classroom, but especially outside the classroom. Please understand what you are getting yourself into. This Summer Assignment should be a lengthy, challenging, yet rewarding and enjoyable review for you. If you go through this packet and have a sense of pride and enjoyment in completing the challenge, you are in the right class and will have a great time learning the material of AP Calculus AB. If on the other hand you are going through this packet and you feel more of a sense of frustration because it is too long, too hard, too tedious, ect… then perhaps AP Calculus AB is not the class for you and perhaps you may consider taking another AP class that aligns more with your interests. Either way the choice is yours. **This assignment is due on the first day of school without exceptions, no late work will be accepted. This assignment will be worth 50% of the first unit's homework grade.**

Sincerely,

Mr. Andrew Masters

In this packet there are three sections of interest:

I. **Things to Know Before the First Day of AP Calculus AB**: A summery of all the things you will be expected to know/do on the first day and on any day/assignment/test/quiz/final.
II. **Summer Assignment: Part I**: The first part of Summer Assignment with an emphasis on graphing various functions and trigonometry.
III. **Summer Assignment: Part II**: The second part of the Summer Assignment with an emphasis on the Rules of Algebra and solving Word Problems.

Things to Know Before the First Day of AP Calculus AB

It will be assumed that students know the following information and can complete the following tasks at the beginning of AP Calculus AB.

Holy Trinity of Algebra: All relations (including functions) can be expressed in three forms: Graphs, sets of solutions/ordered pairs, and equations. Students are able to fluidly move between each of these representations for the following types of functions with or without a calculator:

1. Polynomials (including Power Functions)
2. Rational Functions
3. Algebraic Functions (Created by adding, subtracting, multiplying, dividing, or taking a root of any number of polynomials.)
4. Exponential Functions
5. Logarithmic Functions
6. Trigonometric Functions
7. Inverse Trigonometric Functions

Factoring: Students should be able to factor a polynomial using the following factoring techniques without a calculator. Not every polynomial can be factored using these techniques; you will not be required to factor such polynomials.

1. Factoring a GCF from each term of a polynomial.
2. Grouping
3. Reverse FOIL
4. Difference of Squares
5. Difference of Cubes
6. Sum of Cubes
7. Completing the Square
8. Quadratic Formula
9. $\frac{p}{q}$ Method of Factoring

Functions and their properties: Students should have the following graphs memorized, including a point of interest and any asymptotes. If the function is periodic, students should have the period and amplitude memorized. Students should also be able to give domain and range for each. Students can do this with and without a calculator.

1. $y = x$ Point of interest: (0,0)
2. $y = x^{2n}, n \in \mathbb{N}$ Point of interest: (0,0)
3. $y = x^{2n+1}, n \in \mathbb{N}$ Point of interest: (0,0)
4. $y = \sqrt[2n]{x}, n \in \mathbb{N}$ Point of interest: (0,0)
5. $y = \sqrt[2n+1]{x}, n \in \mathbb{N}$ Point of interest: (0,0)
6. $y = b^x, b > 0, b \neq 1$ Point of interest: (0,1), asymptotes: $y = 0$
7. $y = e^x$ Point of interest: (0,1), asymptotes: $y = 0$
8. $y = \left(\frac{1}{b}\right)^x, b > 0, b \neq 1$ Point of interest: (0,1), asymptotes: $y = 0$

9. $y = \log_b x, b > 0, b \neq 1$ Point of interest: $(1,0)$, asymptotes: $x = 0$
10. $y = \ln x, b > 0, b \neq 1$ Point of interest: $(1,0)$, asymptotes: $x = 0$
11. $y = \log_{\frac{1}{b}} x, b > 0, b \neq 1$ Point of interest: $(1,0)$, asymptotes: $x = 0$
12. $y = \frac{1}{x}$ Point of interest$(0,0)$, asymptotes: $y = 0$ and $x = 0$.
13. $y = \sin\ x$ Point of interest $(0,0)$, period: 2π, amplitude: 1
14. $y = \cos\ x$ Point of interest $(0,1)$, period: 2π, amplitude: 1
15. $y = \tan\ x$ Point of interest $(0,0)$, vert. asymptotes: where $\cos x = 0$, period: π
16. $y = \csc\ x$ Point of interest $(0,0)$, vert. asymptotes: where $\sin x = 0$, period:2π
17. $y = \sec\ x$ Point of interest $(0,0)$, vert. asymptotes: where $\cos x = 0$, period: 2π
18. $y = \cot\ x$ Point of interest $(0,0)$, vert. asymptotes: where $\sin x = 0$, period: π
19. $y = \arcsin\ x$ Point of interest $(0,0)$
20. $y = \arccos\ x$ Point of interest $(0,1)$
21. $y = \arctan\ x$ Point of interest $(0,0)$, asymptotes: $y = \pm\frac{\pi}{2}$

Transformations: Students should have the following transformations memorized. Given an equation, students should be able to graph any type of function above using the following transformations. Given a graph of any of the above functions, students should be able to derive the equation using these transformations. These should be done with and without a calculator.

Suppose $c > 0$:

$y = f(x) + c$, shift the graph $y = f(x)$ a distance of c units upward.
$y = f(x) - c$, shift the graph $y = f(x)$ a distance of c units downward.
$y = f(x - c)$, shift the graph $y = f(x)$ a distance of c units to the right.
$y = f(x + c)$, shift the graph $y = f(x)$ a distance of c units to the left.
$y = cf(x)$, stretch the graph $y = f(x)$ vertically by a factor of c.
$y = \frac{1}{c}f(x)$, compress the graph of $y = f(x)$ vertically by a factor of c.
$y = f(cx)$, compress the graph $y = f(x)$ horizontally by a factor of c.
$y = f\left(\frac{x}{c}\right)$, stretch the graph $y = f(x)$ horizontally by a factor of c.
$y = -f(x)$, reflect the graph $y = f(x)$ about the x −axis.
$y = f(-x)$, reflect the graph $y = f(x)$ about the y −axis.

Linear Functions: Students are familiar with the following formulas associated with lines. Students can work with linear functions with and without a calculator.

1. Slope: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
2. Point-Slope Form: $y - y_1 = m(x - x_1)$
3. Slope Intercept Form: $y = mx + b$
4. x −Intercept: $(a, 0)$

5. y −Intercept: $(0, b)$

Quadratic Functions: Students are familiar with the following formulas associated with Quadratic Functions. Students can work with quadratic functions with and without a calculator.

1. Descending Form (sometimes called standard form): $f(x) = ax^2 + bx + c$
2. Standard Form (sometimes called vertex form): $f(x) = a(x - h)^2 + k$
3. Vertex: (h, k)
4. Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Polynomial Functions: Given any set of polynomial functions, students can determine the following. Students can work with polynomial functions with and without a calculator.

1. Find the Product, Sum, Difference, Quotient, and any roots of the set of polynomials.
2. Students can determine the domain and range.
3. The degree of the polynomial.
4. The maximum number of roots the polynomial has.
5. Irrational roots occur in conjugate pairs.
6. Imaginary roots occur in conjugate pairs.
7. If the polynomial is reasonably factorable (see *Factoring* above), students can determine what intervals the polynomial will yield positive and negative values.
8. End behavior of the polynomial.

Rational Functions: Given any set of rational functions, students can determine the following. Students can work with rational functions with and without a calculator.

1. Find the Product, Sum, Difference, Quotient, and any roots of the set of rational functions.
2. Students can determine the domain and range of a rational function.
3. If the numerator and denominators of the rational functions reasonably factorable (see *Factoring* above), students can simplify the rational function.
4. If the numerator and denominators of the rational functions reasonably factorable (see *Factoring* above), students can determine what intervals the polynomial will yield positive and negative values.
5. End behavior of the rational function.

Radical Functions: Given any set of rational functions, students can determine the following. Students can work with rational functions with and without a calculator.

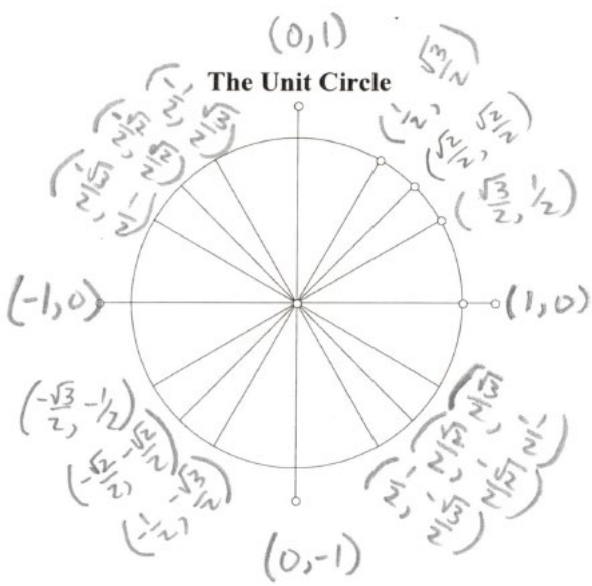
1. Find the Product, Sum, Difference, Quotient, and any roots of the set of radical functions.
2. Students can determine the domain and range of a radical function.
3. If the radicand of the radical function is reasonably factorable (see *Factoring* above), students can simplify the radical function.
4. If the radicand of the radical function is reasonably factorable (see *Factoring* above), students can determine what intervals the polynomial will yield positive and negative values.
5. If a rational function has a radical function as the denominator, students are able to rationalize the denominator.

6. End behavior of the rational function.

Trigonometric Formulas: Students have the following formulas memorized:

1. $\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$
2. $\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$
3. $\tan\theta = \frac{opp}{adj} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$
4. $\csc\theta = \frac{hyp}{opp} = \frac{r}{y} = \frac{1}{\sin\theta}$
5. $\sec\theta = \frac{hyp}{adj} = \frac{r}{x} = \frac{1}{\cos\theta}$
6. $\cot\theta = \frac{adj}{opp} = \frac{x}{y} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$
7. $\sin^2\theta + \cos^2\theta = 1$
8. $1 + \cot^2\theta = \csc^2\theta$
9. $1 + \tan^2\theta = \sec^2\theta$

The Unit Circle: Students have the following information about the Unit Circle memorized.



m^R	m^θ	cos θ	sin θ	tan θ	sec θ	csc θ	cot θ
0	0°	1	0	0	1	undef.	undef.
π/6	30°	√3/2	1/2	√3/3	2√3/3	2	√3
π/4	45°	√2/2	√2/2	1	√2	√2	1
π/3	60°	1/2	√3/2	√3	2	2√3/3	√3/3
π/2	90°	0	1	undef.	undef.	1	0
2π/3	120°	−1/2	√3/2	−√3	−2	2√3/3	−√3/3
3π/4	135°	−√2/2	√2/2	−1	−√2	√2	−1
5π/6	150°	−√3/2	1/2	−√3/3	−2√3/3	2	−√3
π	180°	−1	0	0	−1	undef.	undef.
7π/6	210°	−√3/2	−1/2	√3/3	−2√3/3	−2	√3
5π/4	225°	−√2/2	−√2/2	1	−√2	−√2	1
4π/3	240°	−1/2	−√3/2	√3	−2	−2√3/3	√3/3
3π/2	270°	0	−1	undef.	undef.	−1	0
5π/3	300°	1/2	−√3/2	−√3	2	−2√3/3	−√3/3
7π/4	315°	√2/2	−√2/2	−1	√2	−√2	−1
11π/6	330°	√3/2	−1/2	−√3/3	2√3/3	−2	−√3

Trigonometric Identities:

$$\sin(2x) = 2\sin x\cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos(2x) = 2\cos^2 x - 1$$

$$\cos(2x) = 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sec x = \dfrac{1}{\cos x}$

$\csc x = \dfrac{1}{\sin x}$

$\sin(-x) = -\sin(x)$

$\cos(-x) = \cos(x)$

$\tan(-x) = -\tan(x)$

$\cot(-x) = -\cot(x)$

$\sec(-x) = \sec(x)$

$\csc(-x) = -\csc(x)$

Summer Assignment: Part I

Format Instruction for the Summer Assignment: Please complete both parts of the Summer Assignment on separate sheets of paper. Please complete all problems in order. Please start a problem at the top left of the first page and then start the next problem right below the first problem. Please do not go back to the top of the page, but instead turn the page over or get another sheet of paper and continue the assignment. There should never be two problems occupying the same horizontal space of a sheet of paper. All work must be shown and should be neat and easy to read, all answers should be clearly labeled. All graphs should be done neatly with adequate information (labeling points, axis, etc…). Please staple all pages together and in order and turn it into me on the first day of class. **Failure to follow these formatting instructions will result in a deduction of points. No late work will be accepted.**

For Questions 1-10, graph the parent function and all the given transformations. You may use your calculator to help you; however, ultimately you should be able to graph any of these functions without the use of a calculator.

1) Parent Function: $y = x^2$

a) $y = x^2 - 5$

b) $y = x^2 + 3$

c) $y = (x-10)^2$

d) $y = (x+8)^2$

e) $y = 4x^2$

f) $y = 0.25x^2$

g) $y=-x^2$

h) $y=-(x+3)^2+6$

I) $y=(x+4)^2-8$

j) $y=-2(x+1)^2+4$

2) Parent Function: $y=\sin x$ (set mode to RADIANS)

a) $y=\sin(2x)$

b) $y=\sin x-2$

c) $y=2\sin x$

d) $y=2\sin(2x)+2$

3) Parent Function: $y=\cos x$

a) $y=\cos(3x)$

b) $y=\cos\left(\frac{x}{2}\right)$

c) $y=2\cos x+2$

d) $y=-2\cos x-1$

4) Parent Function: $y=x^3$

a) $y=x^3+2$

b) $y=-x^3$

b) $y=x^3-5$

c) $y=-x^3+3$

e) $y=(x-3)^3$

f) $y=(x-1)^3-4$

g) $y=-2(x+2)^3+1$

h) $y=-\tfrac{1}{3}(x-1)^3+2$

5) Parent Function: $y=\sqrt{x}$

a) $y=\sqrt{x}-2$

b) $y=\sqrt{-x}$

c) $y=\sqrt{x}+5$

d) $y=\sqrt{6-x}$

e) $y=-\sqrt{x}$

f) $y=-\sqrt{-x}$

g) $y=\sqrt{x+2}$

h) $y=\sqrt{2x-6}$

i) $y=-2\sqrt{x}$

j) $y=-\sqrt{4-x}$

6) Parent Function: $y = \ln(x)$

a) $y = \ln(x+3)$

b) $y = \ln(x)+3$

c) $y = \ln(x-2)$

d) $y = \ln(-x)$

e) $y = -\ln(x)$

f) $y = \ln(|x|)$

g) $y = \ln(2x)-4$

h) $y = -3\ln(x)+1$

7) Parent Function: $y = e^x$

a) $y = e^{2x}$

b) $y = e^{x-2}$

c) $y = e^{2-x}$

d) $y = e^{2x}+3$

e) $y = -e^x$

f) $y = e^{-x}$

g) $y = 2-e^x$

h) $y = e^{0.5x}$

8) Parent Function $y = a^x$

a) $y = 5^x$

b) $y = 2^x$

c) $y = 3^{-x}$

d) $y = \left(\frac{1}{2}\right)^x$

e) $y = 4^{x-3}$

f) $y = 2^{x-3}+2$

9) Parent Function: $y = \dfrac{1}{x}$

a) $y = \dfrac{1}{x-2}$

b) $y = \dfrac{-1}{x}$

c) $y = \dfrac{1}{x+4}$ d) $y = \dfrac{2}{5-x}$

10) Parent Function: $y = [x]$

Note: [x] is the IntegerPart of x.

a) $y = [x] + 2$ b) $y = [x-3]$

c) $y = [3x]$ d) $y = [0.25x]$

e) $y = 3 - [x]$ e) $y = 2[x] - 1$

11) Resize your viewing window to [0,1] x [0,1]. <u>Graph all of the following functions in the same window.</u> List the functions from the highest graph to the lowest graph. How do they compare for values of $x > 1$?

a) $y = x^2$ b) $y = x^3$

c) $y = \sqrt{x}$ d) $y = x^{2/3}$

e) $y = |x|$ f) $y = x^4$

For questions 12-19, use your graphing calculator to find the answers.

12) Given: $f(x) = x^4 - 3x^3 + 2x^2 - 7x - 11$

Find all roots to the nearest 0.001

13) Given: $f(x) = 3\sin(2x) - 4x + 1$ on $[-2\pi, 2\pi]$

Find all roots to the nearest 0.001.

Note: All trig functions are done in radian mode.

14) Given: $f(x) = 0.7x^2 + 3.2x + 1.5$

Find all roots to the nearest 0.001.

15) Given: $f(x) = x^4 - 8x^2 + 5$

Find all roots to the nearest 0.001.

16) Given: $f(x) = x^3 + 3x^2 - 10x - 1$

Find all roots to the nearest 0.001

17) Given: $f(x) = 100x^3 - 203x^2 + 103x - 1$

Find all roots to the nearest 0.001

18) Given: $f(x) = |x-3| + |x| - 6$

Find all roots to the nearest 0.001

19) Given: $f(x) = |x| - |x-6|$

Find all roots to the nearest 0.001

Solve the following inequalities, you should be able to do this without the use of a calculator.

20) $x^2 - x - 6 > 0$

21) $x^2 - 2x - 5 \geq 3$

22) $x^3 - 4x < 0$

For each of the following (problems 23-26).

a) Sketch the graph of $f(x)$

b) Sketch the graph of $|f(x)|$

c) Sketch the graph of $f(|x|)$

d) Sketch the graph of $f(2x)$

e) Sketch the graph of $2f(x)$

You may use your calculator to help you; however, ultimately you should be able to graph any of these functions without the use of a calculator.

23) $f(x) = 2x + 3$

24) $f(x) = x^2 - 5x - 3$

25) $f(x) = 2\sin(3x)$

26) $f(x) = -x^3 - 2x^2 + 3x - 4$

For 27-30, You may use your calculator to help you; however, ultimately you should be able to graph any of these functions without the use of a calculator.

27) Let $f(x) = \sin x$

Let $g(x) = \cos x$

a) Sketch the graph of f^2

b) Sketch the graph of g^2

c) Sketch the graph of $f^2 + g^2$

28) Given: $f(x) = 3x + 2$

$g(x) = -4x - 2$

Find the point of intersection

29) Given: $f(x) = x^2 - 5x + 2$

$g(x) = 3 - 2x$

Find the coordinates of any points of intersection.

30) How many times does the graph of $y = 0.1x$ intersect the graph of $y = \sin(2x)$?

You may use a graphing calculator to answer 31.

31) If $f(x) = x^4 - 7x^3 + 6x^2 + 8x + 9$

a) Find the coordinates of the lowest point on the graph.

b) Find the coordinates of the highest point on the graph.

32) Prove the following trigonometric identities by showing that the left side is equal to the right side.

1. $\sin\theta = \cos\theta\tan\theta$

2. $\dfrac{1}{\cos\theta} = \dfrac{\tan\theta}{\sin\theta}$

3. $\sin^2\theta - \cos^2\theta = 1 - 2\cos^2\theta$

4. $\tan^2\theta + 1 = \dfrac{1}{\cos^2\theta}$

5. $1 - \cos^2\theta = \sin\theta\cos\theta\tan\theta$

6. $\cos^2\theta\tan^2\theta = \sin^2\theta$

7. $\sin\theta\tan\theta + \cos\theta = \dfrac{1}{\cos\theta}$

8. $\dfrac{\tan^2\theta}{\sin^2\theta} - 1 = \tan^2\theta$

9. $\cos^2\theta(1 + \tan^2\theta) = 1$

10. $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$

11. $\sin^2\theta - \cos^2\theta = 2\sin^2\theta - 1$

12. $1 + \cos^2\theta = 2\cos^2\theta + \sin^2\theta$

13. $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

14. $\sin\theta(1 + \tan\theta) = \tan\theta(\sin\theta + \cos\theta)$

Summer Assignment Part II begins on the next page. Please follow the same formatting instructions you did in Part I.

AP CALCULUS AB SUMMER ASSIGNMENT NAME________________________
Shafiyoon

I. Basic Algebraic Rules

1. Are the following statements true? If not, change them to make them true.

a. $\frac{2k}{2k+4}=\frac{k}{k+4}$ b. $\frac{1}{p+q}=\frac{1}{p}+\frac{1}{q}$ c. $\frac{x+y}{2}=\frac{x}{2}+\frac{y}{2}$

d. $3\left(\frac{a}{b}\right)=\frac{3a}{3b}$ e. $3\left(\frac{a}{b}\right)=\frac{3a}{b}$ f. $3\left(\frac{a+b}{c}\right)=\frac{3a+b}{c}$

II. Complex Fractions & Rational Expressions

2. Simplify.

a. $\frac{\frac{x}{2}}{\frac{x}{4}}$ b. $h\div\frac{x+h}{h}$ c. $\frac{\sqrt{x-2}+\frac{5}{\sqrt{x-2}}}{x-2}$

3. Write as a single fraction with the denominator in <u>factored form</u>.

a. $\frac{7x^2+5x}{x^2+1}-\frac{5x}{x^2-6}$ b. $20\left(\frac{2}{x+1}-\frac{3}{x}\right)$

c. $x(1-2x)^{-\frac{3}{2}}+(1-2x)^{-\frac{1}{2}}$ d. $(3x-2)^{\frac{1}{2}}+x(3x-2)^{-\frac{1}{2}}$ d. $\frac{\frac{2}{x}-3}{1-\frac{1}{x-1}}$

III. Negative and Fractional Exponents

4. Simplify using only positive exponents. Do not rationalize the denominators.

a. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$ b. $\left(\frac{1}{x^{-2}}+\frac{4}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$ c. $\left(\frac{x^{-2}}{y^{-1}}-x\right)^{-3}$

IV. Solving Equations and Factoring

5. Solve for y' in <u>*simplest form*</u>.

a. $xy' + y = 1 + y'$ b. $3y^2y' + 2yy' = 5y' + 2x$ c. $3x^2yy' + 2xy^2 = 2yy'$

6. Solve the quadratic equation. Use any means from algebra: factoring, quadratic formula, graphing. Be sure answers are simplified.

a. $4x^2-21x-18=0$ b. $2x^2-3x+3=0$ c. $x^4-9x^2+8=0$

7. Factor completely (There should be no fractional or negative exponents.)

a. $3x^3+192$ b. $9x^2-3x-2$ c. $2\sqrt{x}-6x^{\frac{3}{2}}$

d. sinx + tanx e. $e^{-x}-xe^{-x}+2x^2e^{-x}$ f. $2x^4 + 5x^3 - 3x^2$

V. Equations of lines

8. Find the equation of the line that passes through the point (2, 4) and is parallel to the line 2x + 3y - 8 = 0.

9. Find the equation of the line that is perpendicular to the line 2x + 3y - 8 = 0 at the point (1, 2).

10. The line with slope 5 that passes through the point (-1, 3) intersects the x-axis at a point. What are the coordinates of this point?

11. What are the coordinates of the point at which the line passing through the points (1, -3) and (-2, 4) intersects the y-axis?

12. Graph the equation $y = x^3 - x$ and answer the following questions.

a. Is the point (3, 2) on the graph?
b. Is the point (2, 6) on the graph?
c. Is the function odd, even or neither?
d. Find the x and y - intercept(s).

VI. Asymptotes and Intercepts

13. Find all intercepts and asymptotes.

a. $y^2 = x^2 - 4x$ b. $y = \dfrac{x^2 + 3x}{(3x+1)^2}$ c. $y = \dfrac{x^2 - 4}{x^2 - x - 12}$

d. $y = \dfrac{3x-1}{2x^2 + x - 6}$

VII. Domain

14. Use interval notation to identify the domain for each of the following functions.

a. $h(x) = \dfrac{1}{4x^2 - 21x - 18}$ b. $k(x) = \sqrt{x^2 - 5x - 14}$ c. $\dfrac{\sqrt[3]{x-6}}{\sqrt{x^2 - x - 30}}$

d. $d(x) = \ln(2x - 12)$

VIII. Graphing Functions

15. Graph the function. (preferably on graph paper)

a. f(x) = $\begin{cases} 1 & x \le 0 \\ -1 & x > 0 \end{cases}$ b. f(x) = $\begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2) \\ -x+3 & [2, \infty) \end{cases}$ c. f(x) = $\sqrt{16 - x^2}$

16. Graph each.

a. $y = \begin{cases} 3 - x & x \le 1 \\ 2x & x > 1 \end{cases}$ b. $y = \begin{cases} 4 - x^2 & x < 1 \\ \dfrac{3}{2}x + \dfrac{3}{2} & 1 \le x \le 3 \\ x + 2 & x > 3 \end{cases}$ c. $y = \dfrac{|x+1|}{x+1}$ d. $y = |x-2| + 3$

IX. Functions

17. Find f(1) - f(5) given f(x) = |x - 3| - 5.
18. Find f(x + 2) - f(2) given f(x) = $x^2 - 3x + 4$.
19. Find f(x + h) for f(x) = $x^2 - 2x - 3$.
20. Find $\frac{f(x+h)-f(x)}{h}$ if f(x) = $8x^2 + 1$.
21. Given f(x) = x - 3 and g(x) = $\sqrt{x}$, complete the following.

a. f(g(x)) = b. g(f(x)) = c. f(f(x)) =

22. Given $f(x) = \frac{1}{x-5}$ and g(x) = $x^2 - 5$, complete the following.

a. f(g(7)) = b. g(f(v)) = c. g(g(x)) =

X. Binomial Theorem

23. Use the Binomial Theorem to expand and simplify the following expressions.

a. $(x + 5)^5$ b. $(3a - 4b)^4$ c. $(2w + 3)^6$

XI. Factor Theorem

24. Use the p over q method and synthetic division to factor the polynomial P(x). Then solve P(x)=0.

a. $P(x) = x^3 + 5x^2 - 2x - 24$ b. $P(x) = x^4 + 5x^3 + 6x^2 - 4x - 8$

XII. Logarithms

25. Condense the expression $2\ln(x-3) + \ln(x+2) - 6\ln x$

26. Express y in terms of x.

a. lny = x + 2 b. lny = 2lnx + ln10 \

c. lny = 4lnx + 3 d. $x = \ln\frac{e^{x^2}}{4y}$

27. Solve for x.

a. $\ln e^3 = x$ b. $\ln e^x = 4$ c. lnx + lnx = 0 d. $e^{\ln 5} = x$

e. ln1 - lne = x f. ln6 + lnx - ln2 = 3 g. ln(x + 5) = ln(x - 1) - ln(x +1)

XIII. Trigonometry

28. Evaluate (without a calculator!!). NO decimals.

a. cos 0 b. sin 0 c. tan $\frac{\pi}{2}$ d. cos $\frac{\pi}{4}$ e. sin $\frac{\pi}{2}$ f. sin π

g. $\sin^{-1}\frac{\sqrt{3}}{2}$ h. $\tan^{-1} 1$ i. $\cos^{-1}\frac{1}{2}$ j. $\sec^{-1}\sqrt{2}$

k. If $\cos\theta = \frac{5}{13}$ and θ is in Quadrant II, Find the all the remaining trig functions.

29. Which of the following expressions are identical?

a. $\cos^2 x$ b. $(\cos x)^2$ c. $\cos x^2$

30. Which of the following expressions are identical?

a. $(\sin x)^{-1}$ b. arcsinx c. $\sin x^{-1}$ d. $\frac{1}{\sin x}$

31. Solve the following for the indicated variable on the interval $[0, 2\pi)$.

a. 3cosx - 1 = 2 b. 2sin(2x) - √3 = 0 c. $\tan^2 x - 1 = 0$ d. $2\sin^2 x + \sin x = 1$

32. Complete the following trig identities

a. $\sin^2 x + \cos^2 x =$ b. $\tan^2 x + 1 =$ c. $\frac{1 - (\sin x + \cos x)^2}{2\sin x}$

XIV. Word Problems

33. Find the surface area of a box of height *h* whose base dimensions are *p* and *q* and satisfies the following conditons.

a. The box is closed.

b. The box has an open top.

c. The box has an open top and a square base with side length *p*.

34. A seven foot ladder, leaning against a wall, touches the wall x feet above the ground. Write an expression in terms of x for the distance from the foot of the ladder to the base of the wall.

35. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x.

36. A police car receives a radio call to catch a vehicle which is speeding down the highway at 80 mph. The police car, which is 12 miles away, drives after it at 108 mph. How long will it take for the police car to catch up?

37. The base of a triangle is 6 cm more than the height. If the area of the triangle is 140 square cm, what is the length of the base?

38. Two trains, the Express and the Commuter, leave the same station at the same time. The Express, which heads north, travels 10 km per hour faster than the Commuter, which goes east. If the trains are 100 km apart after 2 hours, find the speed of each train.

39. The depth, d, of a buoyant object t seconds after plunging into water can be found using the equation $d = -6t^2 + rt$, where r is the velocity at which the object strikes the water. If the object strikes the water at a velocity of 240 feet per second, find the maximum depth the rocket will reach and at what time. When will the rocket surface again?

Find the average rate of change (i.e. slope) for the following functions on the indicated <u>intervals</u>.

40. $f(x) = x^3 - 2x; [0, 4]$ 41. $f(x) = 3\sqrt{x}; [4, 25]$

42. A car travels 420 miles over a period of 210 minutes. Find the average velocity of the car in miles per hour over this time period.

43. On January 1st 2003, the value of a stock was $135 per share. By December 1st 2003, the value of the stock had fallen to $38 per share. What is the average rate of change in the value of the stock in dollars per month?

44. In 1984, the Fizzy Cola company sold 23 million gallons of soda. By 2003, the company was selling 127 million gallons of soda. What is the average rate of change in the number of gallons of soda sold per year?

45. During a recent trip to the store, a car's velocity went from 0 to 60 mph in 20 seconds. What is the average acceleration of the car in miles per hour per hour?