AP Physics 2 Summer assignment 2023/2024

The summer assignment is worth 100 homework points and is due in on first day of school, written on a separate paper in black ink, with clear headings and your name. Scan your document as 1 PDF document so it is ready to upload to Schoology on the first day of school.

Part 1:

AP Physic 2 uses the same textbook as AP Physics 1.

You have access to Mastering Physics course for AP Physic 1 till August 20th. I have also attached a PDF copy of chapters 13 and 14.

Read chapter 13 Static Fluids and 14 Fluids in motion. Try a few questions on each section of the chapters and make notes on the examples and conceptual exercises. This will cover Unit 1 of AP classroom.

Read and outline each section on a separate sheet, make notes on **key vocabulary**, **definitions**, **equations** and **worked examples**.

Part 2:

Use APlus physics .com https://www.aplusphysics.com/courses/ap-1/AP1_Physics.html#ap2

and watch the following videos and make notes of the key points and the examples.

A. Fluids

1 Density and Buoyancy

2 Pressure and Pascal's Principle

3 Continuity Equation for Fluids

4 Bernoulli's Principle

Part 3:

Once you have completed the summer assignment review Chapters 2 to 10, AP Physics 2 requires use of concepts in AP Physic1, you will benefit from reviewing the topics from your notebooks, APlus physics website and Khan Academy.

Parts of AP Physics 1 are assumed as prior knowledge, in particular Kinematics in 1 and 2 dimensions, Newton's laws, vectors work and Energy.

The following website has all the material on AP Physics 1:

http://www.aplusphysics.com/courses/ap-1/AP1_Physics.html

https://www.khanacademy.org/

For information:

AP Physics 2 covers the following chapters 11 to 30.

You may have to renew your subscription to Mastering Physics and include the digital copy of the textbook. Details of how to renew your subscription will be provided the first day of School. There is an option to purchase a paper copy of the book once you have registered for Mastering Physics.

Capstone software is used for labs with sensors, follow the link below to download and install the software by August 11^{th} , you will be given the license key at the start of the course. https://www.pasco.com/products/software/capstone#downloads-panel.

If you have Capstone already installed run the software and check for updates before first day of school and then at least every month.

Static Fluids

- How can a hot air balloon fly for hours?
- Why is it dangerous for scuba divers to ascend from a deep-sea dive too quickly?
- Why, in water, does a 15-g nail sink but a cargo ship float?

The first hot air balloon with a person on board was launched by the Montgolfier brothers in 1783. Hot air is less dense than cold air. By carefully balancing the density of the balloon and its contents with the density of air, the pilot can control the force that the outside cold air exerts on the balloon. What do pressure, volume, mass, and temperature have to do with this force?

BE SURE YOU KNOW HOW TO:

- Draw a force diagram for a system of interest (Section 3.1).
- Apply Newton's second law in component form (Section 4.2).
- Define pressure (Section 12.2).
- Apply the ideal gas law (Sections 12.3 and 12.4).

IN THE PREVIOUS CHAPTER, we constructed the ideal gas model and used it to explain the behavior of gases. The temperature of the gas and the pressure it exerted on surfaces played an important role in the phenomena we analyzed. We ignored the effect of the gravitational force exerted by Earth on the gas particles. This simplification was reasonable, since in most of the processes we analyzed the gases had little mass and occupied a relatively small region of space, like in a piston. In this chapter, our interest expands to include phenomena in which the force exerted by Earth plays an important role. We will confine the discussion to static fluids—fluids that are not moving.

13.1 Density

We are familiar with the concept of density (from Chapter 12). To find the density of an object or a substance, determine its mass m and volume V and then calculate the ratio of the mass and volume:

$$\rho = \frac{m}{V} \tag{13.1}$$

Archimedes (Greek, 287–212 B.C.) discovered how to determine the density of an object of irregular shape. First determine its mass using a scale. Then determine its volume by submerging it in a graduated cylinder with water (Figure 13.1). Finally, divide the mass in kilograms by the volume in cubic meters to find the density in kilograms per cubic meter. Using this method we find that the density of an iron nail is 7860 kg/m^3—relatively large. A gold coin has an even larger density—19,300 kg/m^3. The universe, though, contains much denser objects. For example, the rapidly spinning neutron star known as a pulsar (discussed in Chapter 9) has a density of approximately 10^{18} kg/m^3. Table 13.1 lists the densities of various solids, liquids, and gases.

FIGURE 13.1 Measuring the density of an irregularly shaped object.

1. Measure mass of object.
2. Place the object in water in a graduated cylinder.
3. Measure the volume change of the water. Volume change of water = volume of object.
4. Density = $\rho = m/V$

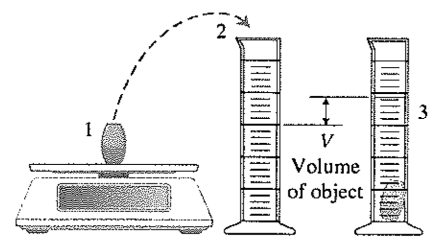


TABLE 13.1 Densities of various solids, liquids, and gases

Solids		Liquids[1]		Gases[2]	
Substance	Density (kg/m^3)	Substance	Density (kg/m^3)	Substance	Density (kg/m^3)
Aluminum	2700	Acetone	791	Dry air 0 °C	1.29
Copper	8920	Ethyl alcohol	789	10 °C	1.25
Gold	19,300	Methyl alcohol	791	20 °C	1.21
Iron	7860	Gasoline	726	30 °C	1.16
Lead	11,300	Olive oil	800–920	Helium	0.178
Platinum	21,450	Mercury	13,600	Hydrogen	0.090
Silver	10,500	Milk	1028–1035	Oxygen	1.43
Bone	1700–2000	Seawater	1025	Carbon dioxide	1.98
Brick	1400–2200	Water 0 °C	999.8		
Cement	2700–3000	3.98 °C	1000.00		
Clay	1800–2600	20 °C	998.2		
Glass	2400–2800	Blood plasma	1030		
Ice	917	Blood whole	1050		
Styrofoam	25–100				
Balsa wood	120				
Oak	600–900				
Pine	500				
Planet Earth	5515				
Moon	3340				
Sun	1410				
Universe (average)	10^{-26}				
Pulsar	10^{11}–10^{18}				

[1]Densities of liquids are at 0 °C unless otherwise noted.
[2]Densities of gases are at 0 °C and 1 atm unless otherwise noted.

QUANTITATIVE EXERCISE 13.1 Ping-pong balls with different densities

Saturn has the lowest density of all the planets in the solar system ($M_{Saturn} = 5.7 \times 10^{26}$ kg and $V_{Saturn} = 8.3 \times 10^{23}$ m^3). The average density of a neutron star is 10^{18} kg/m^3. Compare the mass of a ping-pong ball filled with material from Saturn with that of the same ball filled with material from a neutron star. An empty ping-pong ball has a 0.037-m diameter (0.020-m radius) and a 2.7-g mass.

Represent mathematically To find the mass of a ping-pong ball filled with a particular material, we add the mass of the ball alone and the calculated mass of the material inside:

$$m_{\text{filled ball}} = m_{\text{ball}} + m_{\text{material}}$$

where $m_{\text{material}} = \rho_{\text{material}} V_{\text{ball}}$.

The density of the neutron star is given, and the density of Saturn can be found using the operational definition $\rho_{Saturn} = \dfrac{m_{Saturn}}{V_{Saturn}}$. The interior of the ping-pong ball is a sphere of volume $V_{\text{sphere}} = \frac{4}{3}\pi R^3$. Assume that the plastic shell of the ball has negligible volume. The mass of either filled ball is

$$m_{\text{filled ball}} = m_{\text{ball}} + m_{\text{material}} = m_{\text{ball}} + \rho_{\text{material}} V_{\text{ball}}$$
$$= m_{\text{ball}} + \rho_{\text{material}} \tfrac{4}{3}\pi R_{\text{ball}}^3$$

Solve and evaluate For the neutron-star-filled ball:

$$m_{\text{neutron star ball}} = (0.003\ \text{kg}) + (10^{18}\ \text{kg/m}^3)\tfrac{4}{3}\pi(0.020\ \text{m})^3$$
$$= 3.4 \times 10^{13}\ \text{kg}$$

For the Saturn-filled ball:

$$m_{\text{Saturn ball}} = m_{\text{ball}} + \left(\frac{m_{Saturn}}{V_{Saturn}}\right)\left(\tfrac{4}{3}\pi R_{\text{ball}}^3\right)$$
$$= 0.003\ \text{kg} + \left(\frac{5.7 \times 10^{26}\ \text{kg}}{8.3 \times 10^{23}\ \text{m}^3}\right)\left(\tfrac{4}{3}\pi(0.020\ \text{m})^3\right)$$
$$= 0.003\ \text{kg} + 0.023\ \text{kg} = 0.026\ \text{kg}$$

The material from Saturn has less mass than an equal volume of water. The ball filled with the material from a neutron star has a mass of more than a billion tons!

Try it yourself The mass of the ping-pong ball filled with soil from Earth's surface is 0.050 kg. What is the density of the soil?

Answer 1400 kg/m^3.

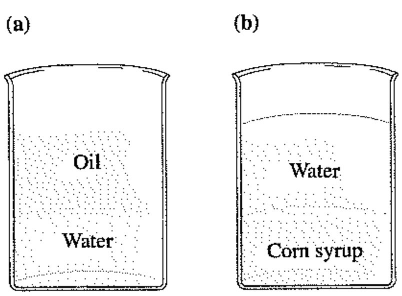


FIGURE 13.2 Less dense matter floats on denser matter.

Density and floating

Understanding density allows us to pose questions about phenomena that we observe almost every day. For example, why does oil form a film on water? If you pour oil into water or water into oil, they form layers (see Figure 13.2a) independently of which fluid is poured first—the layer of oil is always on top of the water. The density of oil is less than the density of water. If you pour corn syrup and water into a container, the corn syrup forms a layer below the water (Figure 13.2b); the density of corn syrup is 1200 kg/m^3, greater than the density of water. Why, when mixed together, is the lower density substance always on top of the higher density substance?

Similar phenomena occur with gases. Helium-filled balloons accelerate upward in air, while air-filled balloons accelerate (slowly) downward. The mass of helium atoms is much smaller than the mass of any other molecules in the air. (Recall that at the same pressure and temperature, atoms and molecules of gas have the same concentration; because helium atoms have much lower mass, their density is lower.) The air-filled balloon must be denser than air. The rubber with which the skin of any balloon is made is denser than air. We can disregard the slight compression of the gas by the balloon, because even though it increases the density of the gas, the effect is the same for both the air and helium in the balloons.

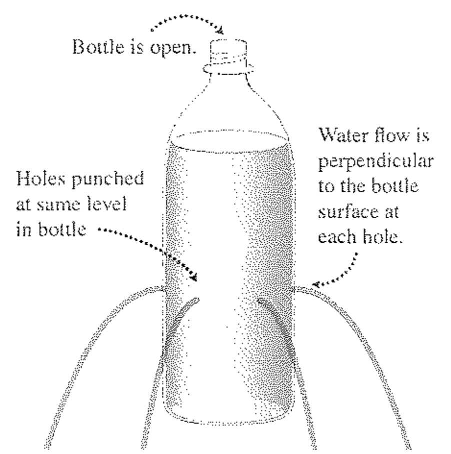
Ice (solid water) floats in liquid water

The solid form of a particular substance is almost always denser than the liquid form of the substance, with one very significant exception: liquid water and solid ice. Since ice floats on liquid water, we can assume that the density of ice is less than that of water. This is in fact true: the density of water changes slightly with temperature and is the highest at 4 °C: 1000 kg/m^3. The density of ice is 917 kg/m^3. Ice has a lower density because in forming the crystal structure of ice, water molecules spread apart. The fact that water expands when it forms ice is important for life on Earth (see the second Reading Passage at the end of this chapter). Fish and plants living in lakes survive cold winters in liquid water under a shield of ice and snow at the surface. Water absorbed in the cracks of rocks freezes and expands in the winter, cracking the rock. Over the years, this process of liquid water absorption, freezing, and cracking eventually converts the rock into soil.

Why do denser forms of matter sink in less dense forms of matter? We learned (in Chapter 12) that the quantity *pressure* describes the forces that fluids exert on each other and on the solid objects they contact. Let us investigate whether pressure explains, for example, why a nail sinks in water or why a hot air balloon rises.

REVIEW QUESTION 13.1 How would you determine the density of an irregularly shaped object?

FIGURE 13.3 Arcs of water leaving holes at the same level in a bottle.

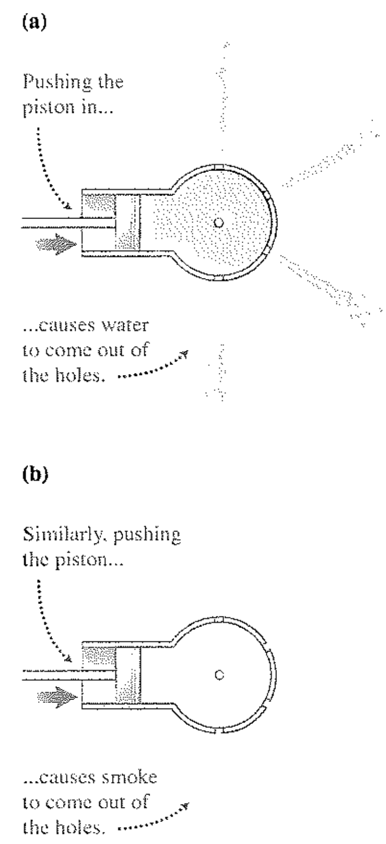


13.2 Pressure inside a fluid

We know that as gas particles collide with the walls of the container in which they reside, they exert pressure. In fact, if you place any object inside a gas, the gas particles exert the same pressure on the object as the gas exerts on the walls of the container. Do liquids behave in a similar way? In the last chapter we learned that the particles in a liquid are in continual random motion, somewhat similar to particles in gases.

Let's conduct a simple observational experiment. Take a plastic water bottle and poke several small holes at the same height along its perimeter. Close the holes with tacks, fill the bottle with water, open the cap, remove the tacks, and observe what happens (Figure 13.3). Identically shaped parabolic streams of water shoot out of the holes. The behavior of the water when the tacks are removed is analogous to a person leaning on a door that is suddenly opened from the other side—the person falls through the door. Evidently, the water inside must push out perpendicular to the wall of the bottle, just as gas pushes out perpendicular to the wall of a balloon. Due to their similar behaviors, liquids and gases are often studied together and are collectively referred to as **fluids**. In addition, since the four streams are identically shaped, the pressure at all points at the same depth in the fluid is the same.

FIGURE 13.4 Pascal's first law: increasing the pressure of a fluid at one location causes a uniform pressure increase throughout the fluid.



Pascal's first law

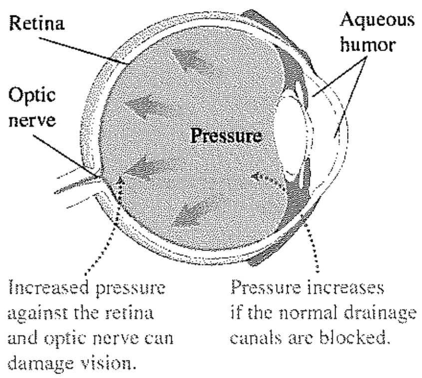
Many practical applications involve situations in which an external object (for example, a piston) exerts a force on a particular part of a fluid. What happens to the pressure at other places inside the fluid? To investigate how the pressures at different points in a fluid are related, we use a special instrument that consists of a round glass bulb with holes in it connected to a glass tube with a piston on the other side (Figure 13.4a). When we fill the apparatus with water and push the piston, water comes out of all of the holes, not only those that align with the piston. When we fill the apparatus with smoke and push the piston, we get the same result (Figure 13.4b). The liquid and the gas behave similarly.

How can we explain this observation? Pushing the piston in one direction caused a greater pressure in the fluid close to the piston. It seems that almost immediately the pressure throughout the fluid increased as well, as the fluid was pushed out of *all* of the holes in the bulb in the same way. This phenomenon was first discovered by French scientist Blaise Pascal in 1653 and is called Pascal's first law.

Pascal's first law **An increase in the pressure of a static, enclosed fluid at one place in the fluid causes a uniform increase in pressure throughout the fluid.**

The above experiment describes Pascal's first law macroscopically. We can also explain Pascal's first law at a microscopic level using gases as an example. Gas particles inside a container move randomly in all directions. When we push harder on one of the surfaces of the container, the gas compresses near that surface. The molecules near

FIGURE 13.5 Glaucoma is an increase in intraocular pressure, caused by blockage of the ducts that normally drain aqueous humor from the eye.



that surface collide more frequently with their neighbors farther from the surface. They in turn collide more frequently with their neighbors. The extra pressure exerted at the one surface quickly spreads, and soon there is increased pressure throughout the gas.

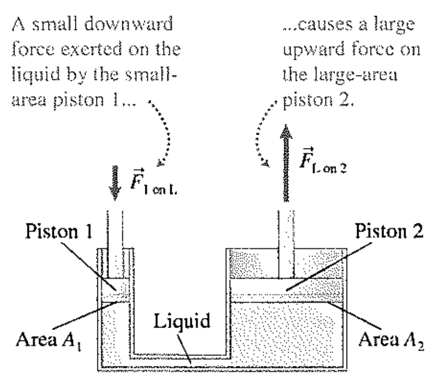
Glaucoma

Pascal's first law can help us understand a common eye problem—*glaucoma*. A clear fluid called *aqueous humor* fills two chambers in the front of the eye (Figure 13.5). In a healthy eye, new fluid is continually secreted into these chambers while old fluid drains from the chambers through sinus canals. A person with glaucoma has closed drainage canals. The buildup of fluid causes increased pressure throughout the eye, including at the retina and optic nerve, which can lead to blindness. Ophthalmologists diagnose glaucoma by measuring the pressure at the front of the eye. The eye pressure of a person with healthy eyes is about $1.2 \times 10^5\ \mathrm{N/m^2}$; a person with glaucoma has an elevated pressure of $1.3 \times 10^5\ \mathrm{N/m^2}$.

Hydraulic lift

One of the technical applications of Pascal's first law is a *hydraulic lift*, a form of simple machine that converts small forces into larger forces, or vice versa. Automobile mechanics use hydraulic lifts to lift cars, and dentists and barbers use them to raise and lower their clients' chairs. The hydraulic brakes of an automobile are also a form of hydraulic lift. Most of these devices work on the simple principle illustrated in Figure 13.6, although the actual devices are usually more complicated in construction.

FIGURE 13.6 Schematic of a hydraulic lift.



In Figure 13.6, a downward force $\vec{F}_{1\text{ on L}}$ is exerted by piston 1 (with small area A_1) on the liquid. This piston compresses a liquid (usually oil) in the lift. The pressure in the liquid just under piston 1 is

$$P_1 = \frac{F_{1\text{ on L}}}{A_1}$$

Because the pressure changes uniformly throughout the liquid, the pressure under piston 2 is also $P = F_{1\text{ on L}}/A_1$, with a key assumption that the pistons are at the same elevation. Since piston 2 has a greater area A_2 than piston 1, the liquid exerts a greater upward force on piston 2 than the downward force on piston 1:

$$F_{\text{L on 2}} = PA_2 = \left(\frac{F_{1\text{ on L}}}{A_1}\right)A_2 = \left(\frac{A_2}{A_1}\right)F_{1\text{ on L}} \tag{13.2}$$

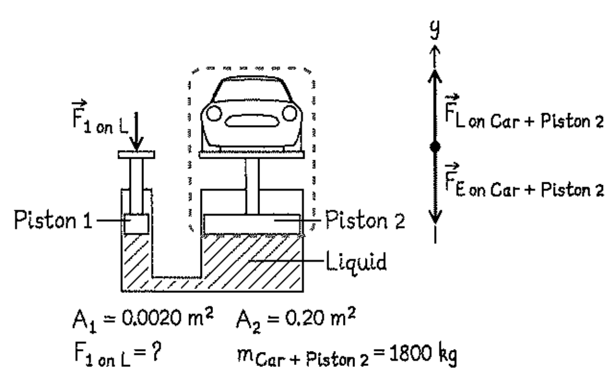
Since A_2 is greater than A_1, the lift provides a significantly greater upward force $F_{\text{L on 2}}$ on piston 2 than the downward push of the smaller piston 1 on the liquid $F_{1\text{ on L}}$.

EXAMPLE 13.2 Lifting a car with one hand

A hydraulic lift similar to that described above has a small piston with surface area $0.0020\ \mathrm{m^2}$ and a larger piston with surface area $0.20\ \mathrm{m^2}$. Piston 2 and the car placed on piston 2 have a combined mass of 1800 kg. What is the minimal force that piston 1 needs to exert on the fluid to slowly lift the car?

Sketch and translate The situation is similar to that shown in Figure 13.6. We need to find $F_{1\text{ on L}}$ so the fluid exerts a force great enough to support the mass of the car and piston 2. The hydraulic lift Eq. (13.2) should then allow us to determine $F_{1\text{ on L}}$.

Simplify and diagram Assume that the levels of the two pistons are the same and that the car is being lifted at constant velocity. Use the force diagram for the car and piston 2 (see diagram at right) and Newton's second law to determine $F_{\text{L on 2}}$. Note that the force that the liquid exerts on the large piston 2 $F_{\text{L on 2}}$ is equal in magnitude to the force that piston 2 and the car exert on the liquid $F_{2\text{ on L}}$, which equals the downward gravitational force that Earth exerts on the car and piston: $F_{2\text{ on L}} = m_{\text{Car+Piston}}g$.



Represent mathematically We rewrite the hydraulic lift Eq. (13.2) to determine the unknown force:

$$F_{1\text{ on L}} = \left(\frac{A_1}{A_2}\right)F_{2\text{ on L}} = \left(\frac{A_1}{A_2}\right)m_{\text{Car+Piston}}g$$

Solve and evaluate

$$F_{1\text{ on L}} = \frac{(0.0020\text{ m}^2)}{(0.20\text{ m}^2)}[(1800\text{ kg})(9.8\text{N/kg})] = 180\text{ N}$$

That is the force equal to lifting an object of mass 18 kg, which is entirely possible for a person. The units are also consistent.

Try it yourself If you needed to lift the car about 0.10 m above the ground, what distance would you have to push down on the small piston, assuming the model of the hydraulic lift applies to industrial lifts? Is the answer realistic, and what does it tell you about the operation of real hydraulic lifts?

Answer

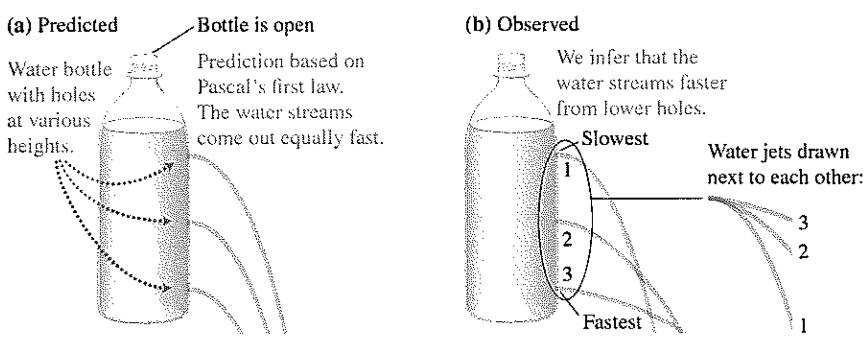
10 m. This answer is too large to be realistic. There must be some other technical details involved in the operation of industrial hydraulic lifts.

REVIEW QUESTION 13.2 If you poke many small holes in a closed toothpaste tube and squeeze it, the paste comes out equally from all holes. Why?

13.3 Pressure variation with depth

Pascal's first law states that an increase in the pressure in one part of an enclosed fluid results in an increase at all other parts of the fluid. Does that mean that the pressure is the same throughout a fluid—for example, in a vertical column of fluid? To test this hypothesis, consider another experiment with a water bottle. This time, poke vertical holes along one side of the bottle. Place tacks in the holes, and fill the bottle with water. Leave the cap off. If the pressure is the same throughout the fluid, when the tacks are removed the water should come out of each hole making an arc of the same shape, similar to projectiles thrown horizontally at the same speed, as shown in Figure 13.7a.

FIGURE 13.7 Water seems to be pushed harder from holes deeper in the water.

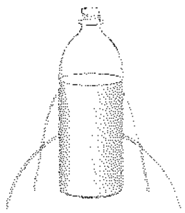
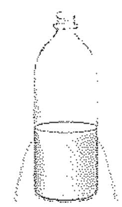
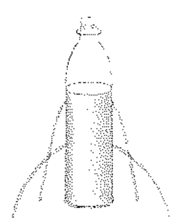


However, when the tacks are removed, we observe that the shapes of the arcs are different. The arcs produced by the water coming out from the lower holes resemble the trajectories of projectiles thrown at higher speeds (Figure 13.7b). Should we abandon Pascal's first law now because the prediction based on it did not match the outcome of the experiment? In such cases, scientists do not immediately throw out the principle but first examine the additional assumptions that were used to make the prediction. In our first experiment with the water bottle, we did not consider the impact of poking holes at different heights. Maybe this was an important factor in the experiment. Let's investigate this in Observational Experiment Table 13.2, on the next page.

OBSERVATIONAL EXPERIMENT TABLE 13.2

How does the location of the holes affect the streams leaving the holes?

VIDEO OET 13.2

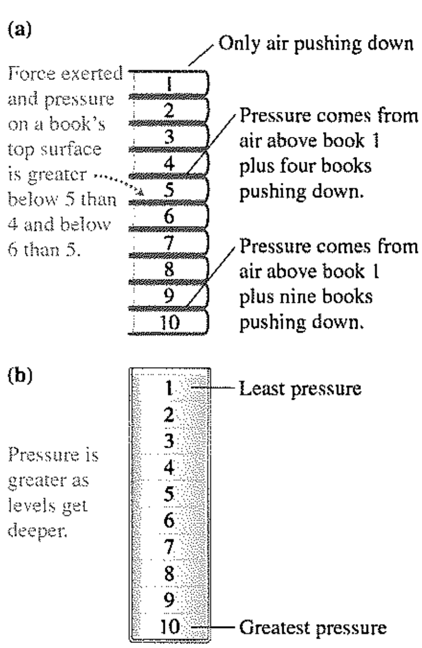
Observational experiment	Analysis
Experiment 1. Place two tacks on each side of a plastic bottle, one hole above the other, and fill the bottle with water above the top tack. Remove the tacks. Water comes out on the left and right, and the stream from the lower holes resembles a projectile thrown at a higher speed. 	There must be greater pressure inside than outside. The pressure must be greater at the bottom holes than at the top holes.
Experiment 2. Repeat Experiment 1 but this time fill the bottle with water to the same distance above the bottom tack as it was filled above the top tack in Experiment 1. Remove the tacks. The stream comes out the bottom holes with the same arc as it came out of the top holes in Experiment 1. 	The total water depth seems not to matter, just the height of the water above the hole.
Experiment 3. Repeat Experiment 1 using a thinner bottle with the water level initially the same distance above the top tack as it was in Experiment 1. Remove the tacks. The water streams are identical to those in Experiment 1. 	Because the water comes out in exactly the same arc in a bigger bottle and in a smaller bottle when the water level above the top tack is at the same height, we can conclude that the mass of the water in the bottle does not affect the pressure.

Patterns

The stream shape at a particular level:

- Depends on the depth of the water above the hole.
- Is the same in different directions at the same level.
- Does not depend on the amount of water (volume or mass) above the hole.
- Does not depend on the amount (mass or volume) or depth of the water below the hole.

FIGURE 13.8 Pressure increases with depth.



From the patterns above we reason that the pressure of the fluid at the hole depends only on the *depth* of the fluid above the hole, and not on the *mass* of the fluid above. We also see that the pressure at a given depth is the same in all directions. This is consistent with the experience you have when you dive below the surface of the water in a swimming pool, lake, or ocean. The pressure on your ears depends only on how deep you are below the surface. Pascal's first law fails to explain this pressure variation at different depths below the surface. Now we need to understand why the pressure varies with depth and to devise a rule to describe this variation quantitatively.

Why does pressure vary at different levels?

To explain the variation of pressure with depth we can use an analogy of stacking ten books on a table (Figure 13.8a). Imagine that each book is a layer of water in a cylindrical tube (see Figure 13.8b). Consider the pressure (force per unit area) from above on the top surface of each book. The only force exerted on the top surface of the top book is due to air pushing down from above. However, there are in effect two forces exerted on the top surface of the second book: the force that the top book exerts on it (equal in magnitude to the weight of the book) plus the force exerted by the air on the top book. The top surface of the bottom book in the stack must balance the force exerted by the nine books above it (equal in magnitude to the weight of nine books) plus the pressure force exerted by the air on the top book. So the pressure increases on the top surface of each book in the stack as we go lower in the stack.

Similar reasoning applies for the fluid-filled tube divided into a number of imaginary thin layers in Figure 13.8b. Air pushes down on the top layer. The second layer balances the weight of the top layer plus the force exerted by the air pushing down on the top layer, and so on. The pressure is lowest at the top of the fluid and greatest at the bottom.

Note that, at each layer, the pressure in a fluid is the same in all directions. If we could take a pressure sensor and place it inside the container of water, the readings of the sensor would be the same independent of the orientation of the sensor as long as its depth remains the same.

How can we quantify pressure change with depth?

We know that pressure increases with depth, and according to our model of pressure using layers, we can hypothesize that it increases with depth linearly (assuming that the density of the fluid remains constant). But what is the slope of the pressure-versus-depth graph, and what is the intercept? Consider the shaded cylinder C of water shown in Figure 13.9a as our system of interest. The walls on opposite sides of the cylinder push inward, exerting equal-magnitude and oppositely directed forces—the forces exerted by the sides cancel. What about the forces exerted by the water above and below? If the pressure at elevation y_2 is P_2 and the cross-sectional area of the cylinder is A, then the fluid above pushes down, exerting a force of magnitude $F_{\text{fluid above on C}} = P_2 A$ (Figure 13.9b). Similarly, fluid from below the shaded section of fluid at elevation y_1 exerts on the cylinder an upward force of magnitude $F_{\text{fluid below on C}} = P_1 A$. Earth exerts a third force on the shaded cylinder $\vec{F}_{\text{E on C}}$ equal in magnitude to $m_C g$, where m_C is the mass of the fluid in the cylinder. Since the fluid is not accelerating, these three forces add to zero. Choosing the y-axis pointing up, we have

$$\Sigma F_y = (-F_{\text{fluid above on C}}) + F_{\text{fluid below on C}} + (-m_C g) = 0$$

Substituting the earlier expressions for the forces, we have

$$(-P_2 A) + P_1 A + (-m_C g) = 0$$

The mass of the fluid in the shaded cylinder is the product of the fluid's density and the volume of the cylinder (assuming the density of the fluid is the same throughout the cylinder):

$$m_C = \rho_{\text{fluid}} V = \rho_{\text{fluid}} [A(y_2 - y_1)]$$

Substituting this expression for the mass in the above expression for the forces, we get

$$-P_2 A + P_1 A - \rho_{\text{fluid}} [A(y_2 - y_1)] g = 0$$

Divide by the common A in all of the terms and rearrange to get

$$P_1 = P_2 + \rho_{\text{fluid}}(y_2 - y_1) g \quad (13.3)$$

This is Pascal's second law. As we see, pressure varies linearly with depth.

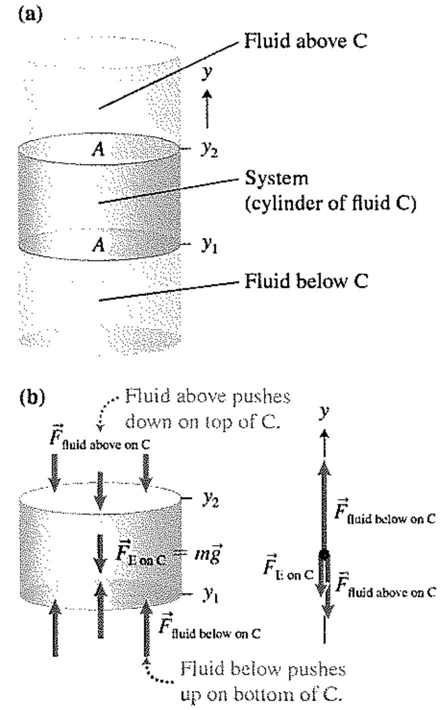


FIGURE 13.9 Using Newton's second law to determine how fluid pressure changes with the depth in the fluid.

Pascal's second law—variation of pressure with depth The pressure P_1 in a static fluid at position y_1 can be determined in terms of the pressure P_2 at position y_2 as follows:

$$P_1 = P_2 + \rho_{\text{fluid}}(y_2 - y_1) g \quad (13.3)$$

where ρ_{fluid} is the fluid density, assumed constant throughout the fluid, and $g = 9.8\ \text{N/kg}$. The positive y-direction is up. If y_2 is chosen to be the top of the fluid, then Eq. (13.3) can be simplified as

$$P_1 = P_{\text{atm}} + \rho_{\text{fluid}} g d \quad (13.4)$$

where P_{atm} is the atmospheric pressure and d is the depth from the top of the fluid to the level at which we want to determine the pressure.

TIP When using Pascal's second law [Eq. (13.3)], picture the situation and be sure to include a vertical y-axis that points upward and has a defined origin, or zero point. Then choose the two points of interest and identify their vertical y-positions relative to the axis. This lets you relate the pressures at those two points.

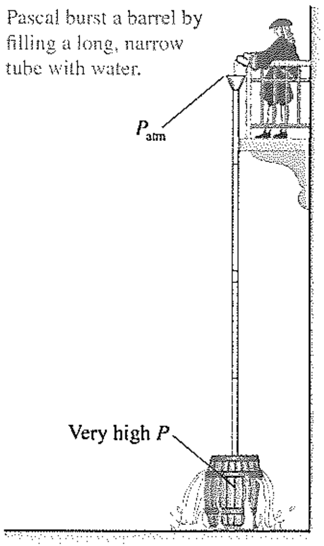


FIGURE 13.10 Pascal tests his second law.

Some history of physics books say that Blaise Pascal conducted the following testing experiment for his second law. Pascal filled a barrel with water and inserted a long, narrow vertical tube into the water from above. He then sealed the barrel (see Figure 13.10). He predicted that when he filled the tube with water, the barrel would burst even though the mass of water in the thin tube was small, because the pressure of the water in the barrel would depend on the depth, not the mass, of the water column. The barrel burst, matching Pascal's prediction, thus supporting the idea that the depth of the fluid above, not its mass determined the pressure.

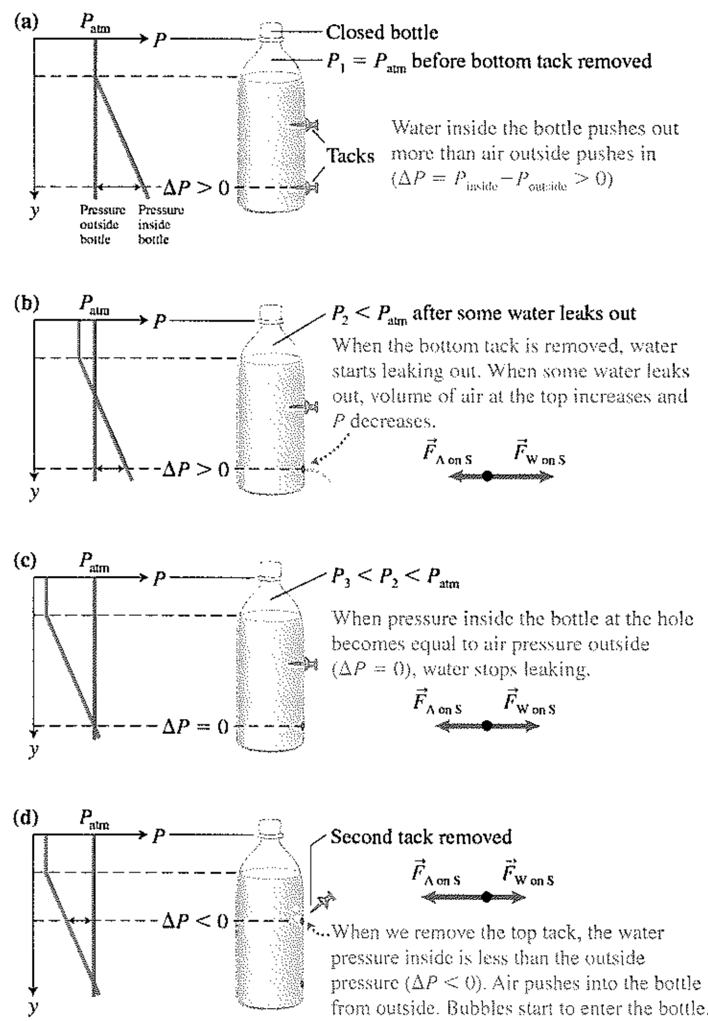
We derived Eqs. (13.3) and (13.4) using liquid as an example. The density of liquids does not change with depth because they are incompressible. Gases are compressible, and thus their density changes with depth. However, in all our examples, the changes in depth will be small enough so that we can neglect the density change in gases. Applying Eq. (13.3) to atmospheric air, we can explain why the pressure at the top of a mountain is less than at the bottom.

CONCEPTUAL EXERCISE 13.3 Closed water bottle with tacks

You have a closed plastic water bottle with two holes in the side closed with tacks, one tack near the bottom of the bottle and the other in the middle. You remove the bottom tack, and a few drops of water come out of the hole but then the leaking stops. Then you remove the top tack (with the bottom hole still open), and an air bubble enters the bottle through the top hole. Draw depth-versus-pressure graphs and force diagrams to explain this phenomenon.

Sketch and translate We sketch the bottle with two tacks in the holes. Let's choose the cap of the bottle to be the origin of the y-axis that points down; the horizontal axis will represent the pressure (Figure a).

Simplify and diagram We consider the pressure above the water level to be atmospheric at the beginning of the experiment (Figure a). Because only a few drops of water leave the bottle, we will neglect the changes in water level on the sketches and graphs. Figures b–d show the removal of the tacks and the changes in pressure on the graphs. We draw force diagrams for a tiny portion of water at the hole as our system of interest. There are two horizontal forces exerted on that system element of water: the force exerted by the outside air pushing inward $\vec{F}_{A\text{ on }S}$ and the force of the inside water pushing outward $\vec{F}_{W\text{ on }S}$. If one of the forces is smaller, then the tiny system of water will either accelerate outward and we will see the bottle leaking, or it will accelerate inward and we will see bubbles of air coming in. When a little bit of water leaks out, the volume of air above the water in the bottle increases, and its pressure decreases. This decrease leads to a decrease of the pressure everywhere inside the bottle.



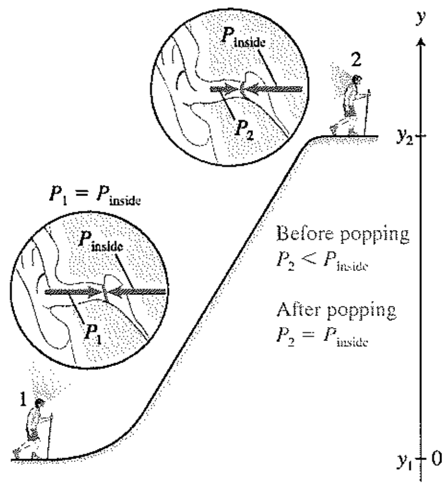
Try it yourself Predict what will happen after an air bubble enters the bottle through the top hole.

Answer

As new air enters the bottle, the air pressure above the water surface increases and so does the pressure elsewhere in the bottle. This results in more water leaking through the lower hole, which again decreases the pressure and causes more bubbles to enter the bottle, and so on. The process repeats until the water level drops to the upper hole. From then on, the water leaks out continuously until it reaches the level of the bottom hole.

QUANTITATIVE EXERCISE 13.4 Pop your ears

If your ears did not pop, then what would be the net force exerted by the inside and outside air on your eardrum at the top of a 1000-m-high mountain? You start your hike from sea level. The area of your eardrum is 0.50 cm^2. The density of air at sea level at standard conditions is 1.3 kg/m^3. Assume the air density remains constant during the hike. The situation at the start at $y_1 = 0$ and at the end of the hike at $y_2 = 1000$ m is sketched below.



Represent mathematically We use an upward-pointing vertical y-axis with the origin at sea level. Assume that the air pressure inside the eardrum remains constant at its sea level value $P_{\text{inside}} = P_1$. The air pressure difference between the top of the mountain and sea level is

$$P_2 - P_1 = \rho(y_1 - y_2)g$$

When at 1000 m above sea level, the outside air exerts a lower pressure P_2 on the eardrum. So the net force exerted by the air on the drum is the pressure difference of air pushing out $P_{\text{inside}} = P_1$ and air pushing in P_2 times the area of the eardrum: $F_{\text{net air on drum}} = (P_1 - P_2)A$. This pressure difference can be determined using Pascal's second law.

Solve and evaluate

$$\begin{aligned} P_1 - P_2 &= \rho(y_2 - y_1)g \\ &= (1.3\ \text{kg/m}^3)(1000\ \text{m} - 0)(9.8\ \text{N/kg}) \\ &= +0.13 \times 10^5\ \text{N/m}^2 \end{aligned}$$

This is 0.13 atm!

$$\begin{aligned} F_{\text{net air on drum}} &= (P_1 - P_2)A \\ &= (0.13 \times 10^5\ \text{N/m}^2)(0.50\ \text{cm}^2) \times (1\ \text{m}/100\ \text{cm})^2 \\ &= +0.60\ \text{N} \end{aligned}$$

The net force is exerted outward and is about half the gravitational force that Earth exerts on an apple. No wonder it can hurt until you get some air out of that middle ear!

Try it yourself Determine the difference in water pressure on your ear when you are 1.0 m underwater compared to when you are at the surface. The density of water is 1000 kg/m^3.

Answer +9800 N/m^2 greater pressure when under the water, or 0.1 atm.

REVIEW QUESTION 13.3 Pascal's first law says that an increase in pressure in one part of an enclosed liquid results in an increase in pressure throughout all parts of that fluid. Why then does the pressure differ at different heights?

13.4 Measuring atmospheric pressure

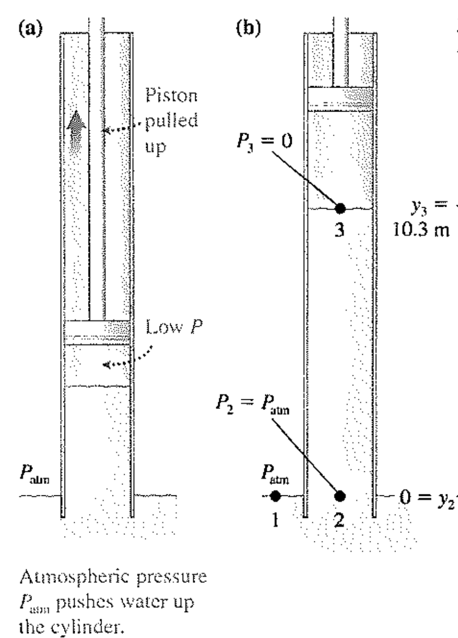
We can now use Pascal's second law to develop a method for measuring atmospheric air pressure.

Torricelli's experiments

In the 1600s, suction pumps were used to lift drinking water from wells and to remove water from flooded mines. The suction pump was like a long syringe. The pump consisted of a piston in a long cylinder that pulled up water (Figure 13.11a). Such pumps could lift water a maximum of 10.3 m. Why 10.3 m?

Evangelista Torricelli (1608–1647), one of Galileo's students, hypothesized that the pressure of the air in the atmosphere could explain the limit to how far water could be lifted. Torricelli did not know of Pascal's second law, which was published in the year that Torricelli died. However, it is possible that Torricelli's work influenced Pascal. Let's analyze the situation shown in Figure 13.11b.

FIGURE 13.11 A piston pulled up a cylinder causes water to rise to a maximum height of 10.3 m.



Consider the pressure at three places: point 1, at the water surface in the pool outside the cylinder; point 2, at the same elevation but inside the cylinder; and point 3, in the cylinder 10.3 m above the pool water level. The pressure at point 1 is atmospheric pressure. The pressure at point 2, according to Pascal's second law, is also atmospheric pressure, since it is at the same level as point 1. To get the water to the 10.3-m maximum height, the region above the water surface inside the cylinder and under the piston must be at the least possible pressure—essentially a vacuum. Thus, we assume that the pressure at point 3 is zero.

Now we will use Eq. (13.3) with $y_3 - y_2 = 10.3$ m to predict the pressure P_2:

$$P_2 = P_3 + \rho_{\text{water}}(y_3 - y_2)g = 0 + (1.0 \times 10^3\,\text{kg/m}^3)(10.3\,\text{m} - 0)(9.8\,\text{N/kg})$$
$$= 1.01 \times 10^5\,\text{N/m}^2$$

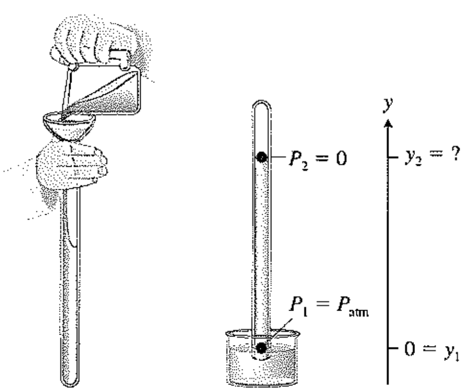
This number is exactly the value of the atmospheric pressure that we encountered in our discussion of gases (in Chapter 12). The atmospheric pressure pushing down on the water outside the tube can push water up the tube a maximum of 10.3 m if there is a vacuum (absence of any matter) above the water in the tube.

At Torricelli's time, the value of normal atmospheric pressure was unknown, so the huge number that came out of this analysis surprised Torricelli. Not believing the result, he tested it using a different liquid—mercury. Mercury is 14 times denser than water ($\rho_{\text{Hg}} = 13{,}600\,\text{kg/m}^3$); hence, the column of mercury should rise only 1/14 times as high in an evacuated tube. However, instead of using a piston to lift mercury, Torricelli devised a method that guaranteed that the pressure at the top of the column was about zero. Consider Testing Experiment Table 13.3.

TESTING EXPERIMENT TABLE 13.3 Testing Torricelli's hypothesis using mercury

Testing experiment

- Torricelli filled a long glass tube closed at one end with mercury.
- He put his finger over the open end and placed it upside down in a dish filled with mercury. He then removed his finger.



Predict what he observed based on the hypothesis that atmospheric pressure limits the height of the liquid in a suction pump.

Prediction

Mercury should start leaking from the tube into the dish. When it leaks, it leaves an empty evacuated space at the top of the tube. It will leak until the height of the mercury column left in the tube produces the same pressure as the atmosphere at the bottom of the column at position 1. The height of the mercury in the tube should be

$$y_2 - y_1 = \frac{P_1 - P_2}{\rho_{\text{mercury}}g}$$
$$= \frac{(1.01 \times 10^5\,\text{N/m}^2 - 0)}{(13.6 \times 10^3\,\text{kg/m}^3)(9.8\,\text{N/kg})}$$
$$= 0.76\,\text{m}$$

Outcome

Torricelli observed some mercury leaking from the tube and then the process stopped. He measured the height of the remaining mercury to be 0.76 m = 760 mm, in agreement with the prediction.

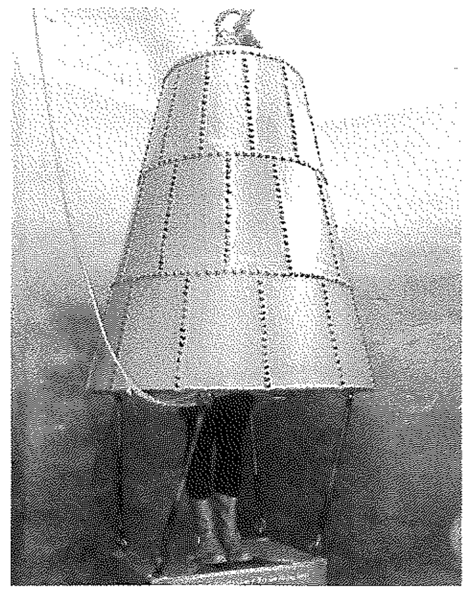
Conclusion

The outcome of the experiment was consistent with the prediction based on Torricelli's hypothesis that atmospheric pressure limits the height of the liquid being lifted in a suction pump. Thus the hypothesis is supported by evidence.

Torricelli also used his understanding of pressure and fluids to predict that in the mountains, where the atmospheric pressure is lower, the height of the mercury column should be lower. Experiments have shown that the mercury level indeed decreases at higher elevation. These experiments supported the explanation that atmospheric air pushes the liquids upward into the tubes.

Torricelli's apparatus with the mercury tube became a useful device for measuring atmospheric pressure—called a barometer. However, since mercury is toxic, the Torricelli device has since been replaced by the aneroid barometer (described in Chapter 12).

Eugenia, one of the book's authors, stands under a 17th-century diving bell displayed in the Vasa Museum in Stockholm, Sweden.



TIP We now understand why pressure is often measured and reported in mm Hg and why atmospheric pressure is 760 mm Hg. The atmospheric pressure (101,000 N/m^2) can push mercury (density 13,600 kg/m^3) 760 mm up a column.

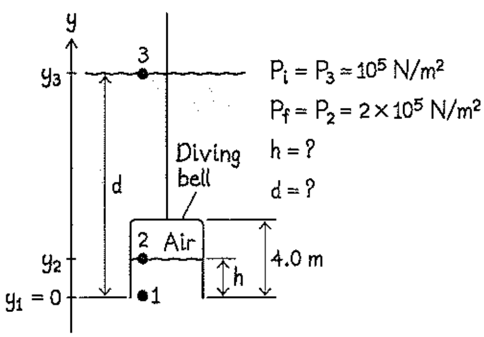
Diving bell

Our understanding of atmospheric pressure allows us to explain many simple experiments that lead to important practical applications. For example, have you ever submerged a transparent container upside down under water? If you have, you have seen that at first little water enters the container; then as the inverted container is pushed deeper into the water, more water enters the container. One practical application of this phenomenon is a diving bell—a large, bottomless chamber lowered under water with people and equipment inside. Divers use the diving bell to take a break and refill on oxygen. In the past, diving bells were used for underwater construction, such as building bridge foundations.

EXAMPLE 13.5 **Diving bell**

The bottom of a 4.0-m-tall cylindrical diving bell is at an unknown depth underwater. The pressure of the air inside the bell is $2.0 \times 10^5\ N/m^2$ (having been about $1.0 \times 10^5\ N/m^2$ before the bell entered the water). The density of ocean water is slightly higher than fresh water: $\rho_{ocean\ water} = 1027\ kg/m^3$. How high is the water inside the bell, and how deep is the bottom of the bell under the water?

Sketch and translate A labeled sketch shows the situation. We want to find the height h of the water in the bell and the depth d that the bottom of the bell is under the water.



Simplify and diagram Assume that the temperature is constant so we can apply our knowledge of an isothermal process (PV is constant for a constant temperature gas) to the air inside the bell and use it to relate the state of the air inside the bell before it is submerged to its state after it is submerged. This will let us find the ratio of the air volumes before and after, and from that we can determine h. Once we have that result, we can use Pascal's second law to determine d.

Represent mathematically Apply the mathematical expression for an isothermal process to the process of submerging the bell, where the initial state is just before the bell starts to be submerged in the water and the final state is when submerged to some unknown depth. We have

$$P_3 V_i = P_2 V_f \quad \Rightarrow \quad V_f = \frac{P_3}{P_2} V_i$$

Next, use Pascal's second law to determine d. The bottom of the bell is at $y_1 = 0$; the water surface is at $y_3 = d$. Compare the pressure at the ocean surface P_3 to the pressure at the water surface inside the bell P_2:

$$P_2 = P_3 + \rho(y_3 - y_2)g$$

Solve and evaluate $V_f = \frac{P_3}{P_2} V_i = \frac{1.0 \times 10^5\ N/m^2}{2.0 \times 10^5\ N/m^2} V_i = \frac{1}{2} V_i$

Thus, the air volume inside the diving bell is half of what it was before entering the water. The bell is therefore half full of water, which means the height of the water level inside is

$$h = \frac{1}{2}(4.0\ m) = 2.0\ m = y_2$$

(CONTINUED)

Rearrange Pascal's second law to solve for the position y_2 of the water level inside the bell:

$$y_2 = 2.0\text{ m} = y_3 - \frac{P_2 - P_3}{\rho g} = d - \frac{(2.0 \times 10^5\text{ Pa} - 1.0 \times 10^5\text{ Pa})}{(1027\text{ kg/m}^3)(9.8\text{ N/kg})}$$

$$= d - 10\text{ m}$$

Thus, $d = 2.0\text{ m} + 10\text{ m} = 12\text{ m}$. The position of the bottom of the bell is 12 m below the ocean's surface.

Try it yourself Suppose the air pressure in the bell is 3.0 atm. In this case, how high is the water in the bell, and how deep is the bottom of the bell?

Answer The water is 2.7 m high in the bell (there is 1.3 m of air at the top), and the bottom of the bell is 23 m below the ocean's surface.

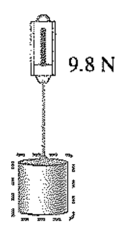
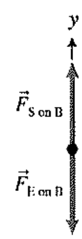
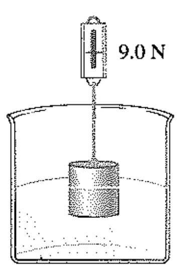
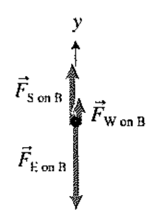
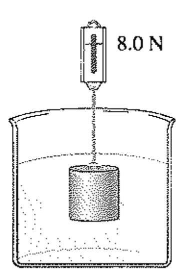
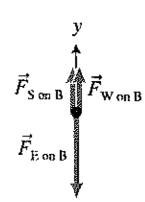
REVIEW QUESTION 13.4 What does it mean if atmospheric pressure is 760 mm Hg?

13.5 Buoyant force

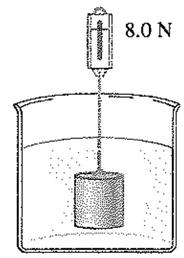
Pascal's first law tells us that pressure changes in one part of a fluid result in pressure changes in other parts. Pascal's second law describes how the pressure in a fluid varies depending on the depth in the fluid. Do these laws explain why some objects float and others sink? Consider Observational Experiment Table 13.4.

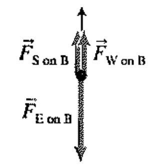
OBSERVATIONAL EXPERIMENT TABLE 13.4 Effect of depth of submersion on a steel block suspended in water

VIDEO OET 13.4

Observational experiment	Analysis
Experiment 1. Hang a 1.0-kg block from a spring scale. The force that the scale exerts on the block balances the downward force that Earth exerts on the block ($mg = (1.0\text{ kg})(9.8\text{ N/kg}) = 9.8\text{ N}$). 	Force diagram for the block B: 
Experiment 2. Lower the block into a container of water, so it is partially submerged. The water level rises. The reading of the scale decreases. 	We explain the decreased reading on the scale by the water pushing upward a little on the block. 
Experiment 3. Lower the same block into the container of water to the point where the block is completely submerged. As the water level rises, the reading of the scale decreases. 	The upward force exerted by the water increases. 

Observational experiment	Analysis
Experiment 4. Lower the block into the container of water so that the block is completely submerged near the bottom. The water level and the reading of the scale do not change.	The upward force exerted by the water does not change once it is completely submerged.





Patterns

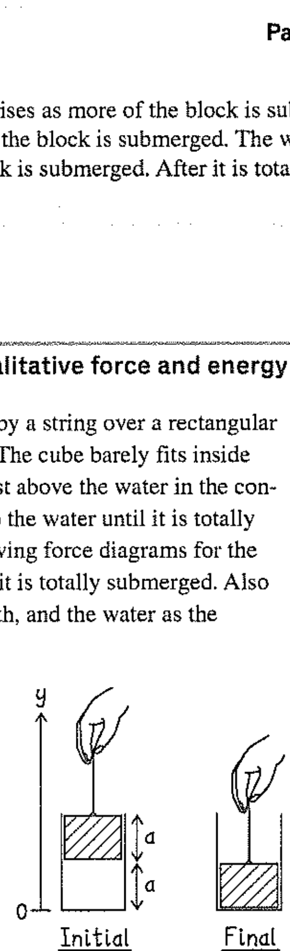
We notice two effects:

1. The level of the water in the container rises as more of the block is submerged in the water.
2. The scale reading decreases as more of the block is submerged. The water exerts an upward force on the block. The magnitude of this force depends on how much of the block is submerged. After it is totally submerged, the force does not change, even though the depth of submersion changes.

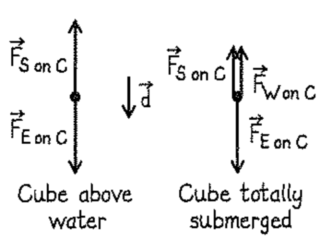
CONCEPTUAL EXERCISE 13.6 Qualitative force and energy analysis for an object in a fluid

You hang a solid metal cube (with side a) by a string over a rectangular container filled with water up to height a. The cube barely fits inside the container. Starting when the cube is just above the water in the container, you very slowly lower the cube into the water until it is totally submerged. Represent this process by drawing force diagrams for the cube when it is above the water and when it is totally submerged. Also draw an energy bar chart for the cube, Earth, and the water as the system for those states.

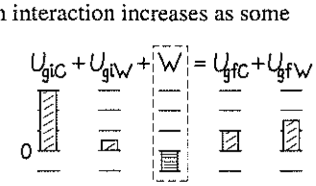
Sketch and translate We draw a sketch of the situation; the system and the initial and final states are specified in the problem statement. The submerged cube displaces water that is now above the cube. The volumes of the cube and water are the same, but the water has less mass due to its smaller density.



Simplify and diagram Because you are moving the cube very slowly, we can neglect the kinetic energy of the cube and the water in this process. We also assume that the cube has no acceleration during this motion and all friction effects are negligible. On the force diagrams we show the three forces exerted by the three objects with which the cube interacts: Earth, the water, and the string. Because there is no acceleration, the sum of the forces is zero.

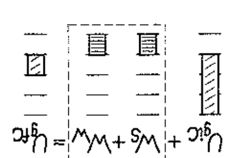


The string does negative work on the cube because the force it exerts on the cube points opposite to the cube's displacement. Given that both water and the cube interact with Earth, we consider the gravitational potential energies of these interactions separately. We take zero gravitational potential energy to be at the bottom of the container. In the initial state, the system has initial gravitational potential energy due to the location of the cube and water. The gravitational potential energy of cube-Earth interaction decreases as the cube moves down; the gravitational potential energy of water-Earth interaction increases as some water moves up (being replaced by the cube). However, the total gravitational potential energy of the system must decrease due to the negative work done by the string.



Try it yourself Draw the bar charts for the two states, choosing the cube and Earth as a system.

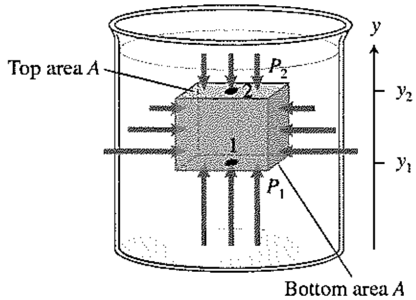
Answer



In Table 13.4, after the block is completely submerged, the scale reads 8.0 N instead of 9.8 N. Evidently, the water exerts a 1.8-N upward force on the block. What is the mechanism responsible for this force?

FIGURE 13.12 A fluid exerts an upward buoyant force on the block.

(a) The force exerted on the block by the fluid below is greater than the force exerted by the fluid above.



(b) The upward force of the fluid on the bottom surface is greater than the downward force of the fluid on the top surface.

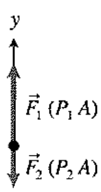
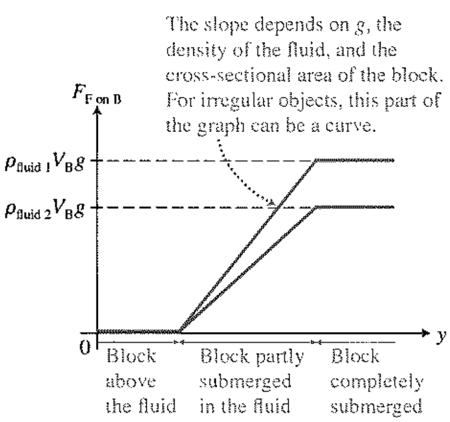


FIGURE 13.13 Buoyant force dependence on the depth of the object and the density of the fluid.



TIP The derivation of Eq. (13.5) was for a solid cube, but the result applies to objects of any shape, though calculus is needed to establish that.

The magnitude of the force the fluid exerts on a submerged object

Consider *only* the fluid forces exerted on the block shown in Figure 13.12a. The fluid pushes inward on the block from all sides, including the top and the bottom. The forces exerted by the fluid on the vertical sides of the block cancel, since the pressure at a specific depth is the same magnitude in all directions.

What about the fluid pushing down on the top and up on the bottom of the block? The pressure is greater at elevation y_1 at the bottom of the block than at elevation y_2 at the top surface of the block. Consequently, the force exerted by the fluid pushing up on the bottom of the block is greater than the force exerted by the fluid pushing down on the top of the block. Arrows in Figure 13.12b represent the forces that the fluid exerts on the top and bottom of the block. The vector sum of these two fluid forces always points up and is called a buoyant force $\vec{F}_{\text{F on B}}$ (fluid on block).

To calculate the magnitude of the upward buoyant force $F_{\text{F on B}}$ exerted by the fluid on the block, we use Eq. (13.3) to determine the pressure P_1 of the fluid on the bottom surface of the block compared to the pressure P_2 of the fluid on the top surface (see Figure 13.12):

$$P_1 = P_2 + \rho_{\text{fluid}}(y_2 - y_1)g$$

The magnitudes of the forces exerted by the fluid on the top and on the bottom of the block are the products of the pressure P and the area A of the top and bottom surfaces of the block:

$$P_1 A = P_2 A + \rho_{\text{fluid}}(y_2 - y_1)Ag$$

or

$$F_1 = F_2 + \rho_{\text{fluid}}(y_2 - y_1)Ag$$

The volume of the block is

$$V_B = A(y_2 - y_1)$$

where A is the cross-sectional area of the block and $(y_2 - y_1)$ is its height. Substitute this volume into the above force equation and rearrange it to get an expression for the magnitude of the total upward **buoyant force** $F_{\text{F on B}}$ that the fluid exerts on the block (the object of interest):

$$F_{\text{F on B}} = F_1 - F_2 = \rho_{\text{fluid}}\, gV$$

Note that for a totally submerged block, V is the volume of the block. However, when it is partially submerged, V is the volume of the submerged part.

We can now understand the results of the experiments in Table 13.4. When submerging the block further into the water, the scale reading decreased because the buoyant force was increasing. The scale reading stopped changing after the block was completely under the water. Once completely underwater, the submerged volume did not change; the upward buoyant force exerted by the fluid on the block remained constant. Fluids of different densities exert different upward forces on the same object submerged to the same depth (Figure 13.13).

Archimedes' principle—the buoyant force A stationary fluid exerts an upward buoyant force on an object that is totally or partially submerged in the fluid. The magnitude of the force is the product of the fluid density ρ_{fluid}, the volume $V_{\text{displaced}}$ of the fluid that is displaced by the object, and the gravitational constant g:

$$F_{\text{F on O}} = \rho_{\text{fluid}} V_{\text{displaced}} g = \rho_{\text{fluid}} V_{\text{subm}} g \qquad (13.5)$$

For simplicity, we will always use the volume of the submerged part of the object in our calculations and label it V_{subm}. Because $\rho_{fluid} V_{displaced} = m_{displaced\ fluid}$, we see that **the buoyant force is equal in magnitude to the force that Earth exerts on the amount of fluid displaced by the object.** However, the natures of these two forces are different. Earth exerts a gravitational force at a distance; the buoyant force is a contact force.

REVIEW QUESTION 13.5 Why does a fluid exert an upward force on an object submerged in it?

13.6 Skills for analyzing static fluid problems

In this section we adapt our problem-solving strategy to analyze processes involving static fluids.

PROBLEM-SOLVING STRATEGY 13.1 Analyzing situations involving static fluids

EXAMPLE 13.7 Buoyant force exerted by air on a human

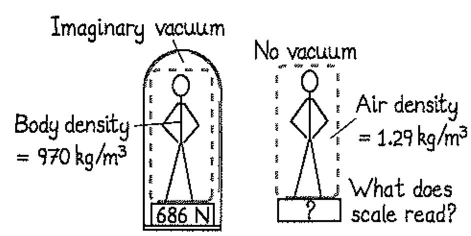
Suppose your mass is 70.0 kg and your density is 970 kg/m^3. If you could stand on a scale in a vacuum chamber on Earth's surface, the reading of the scale would be $mg = (70.0\ kg)(9.80\ N/kg) = 686\ N$. What will the scale read when you are completely submerged in air of density 1.29 kg/m^3?

Sketch and translate

- Make a labeled sketch of the situation and choose the system of interest. If applicable, decide on the initial and final states.
- Include all known information in the sketch and indicate the unknown(s) you wish to determine.

You are the system.

The scale reads 686 N when in a vacuum. Your density is 970 kg/m^3 and the density of air is 1.29 kg/m^3. What does it read when you are submerged in air?

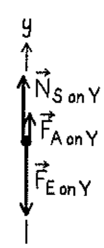


Simplify and diagram

- Indicate any assumptions you are making.
- Identify objects outside the system that interact with it.
- Construct a force diagram for the system, including a vertical coordinate axis. The buoyant force is just one of the forces included in the diagram.
- Construct a bar chart or any other graphical representation that might help solve the problem.

Assume that the air density is uniform.

Three objects exert forces on you. Earth exerts a downward gravitational force $F_{E\ on\ Y} = mg = 686\ N$. The air exerts an upward buoyant force $F_{A\ on\ Y} = \rho_{air} g V_Y$. The scale exerts an unknown upward normal force of magnitude $N_{S\ on\ Y}$.



(CONTINUED)

Represent mathematically

- Use the force diagram to help apply Newton's second law in component form.
- Use the energy bar chart to calculate work and energy if needed.
- Use the expression for the buoyant force and the definitions of pressure and density if needed; sometimes you might need the ideal gas law.

The y-component form of Newton's second law for your body with zero acceleration is (assuming the upward direction as positive)

$$0 = N_{\text{S on Y}} + F_{\text{A on Y}} + (-F_{\text{E on Y}})$$

or

$$N_{\text{S on Y}} = +F_{\text{E on Y}} - F_{\text{A on Y}}$$

The buoyant force that the air exerts on your body has magnitude $F_{\text{A on Y}} = \rho_{\text{air}} V_{\text{Y}} g$. The volume of your body is $V_{\text{Y}} = (m/\rho_{\text{body}})$. The magnitude of the buoyant force that the air exerts on you is

$$F_{\text{A on Y}} = \rho_{\text{air}}\left(\frac{m}{\rho_{\text{body}}}\right)g = mg\left(\frac{\rho_{\text{air}}}{\rho_{\text{body}}}\right)$$

Thus the reading of the scale should be:

$$N_{\text{S on Y}} = mg - mg\left(\frac{\rho_{\text{air}}}{\rho_{\text{body}}}\right)$$

Solve and evaluate

- Insert the known information and solve for the desired unknown.
- Evaluate the final result in terms of units, reasonable magnitude, and whether the answer makes sense in limiting cases.

$$N_{\text{S on Y}} = +686\ \text{N} - (686\ \text{N})\frac{1.29\ \text{kg/m}^3}{970\ \text{kg/m}^3} = 685\ \text{N}$$

According to Newton's third law, the force that you exert on the scale $N_{\text{Y on S}}$ is equal in magnitude to the force the scale exerts on you.

The reading of the scale is actually 0.1% less when you step on the scale in air—not a big deal. We can usually neglect air's buoyant force. Notice that overall the atmospheric air pushes up on objects, not down.

Try it yourself What will the scale read if you weigh yourself in a swimming pool with your body completely submerged?

Answer

0 N. Because you are less dense than water, the buoyant force exerted by the water on you will completely support you, and the scale will not push upward on you at all.

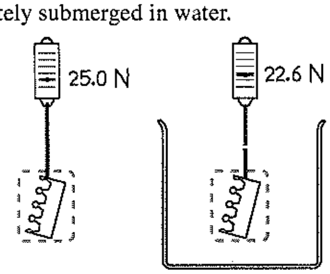
We will use these strategies to analyze several more situations. In the next example, it might not be obvious how to arrive at the answer to the question with the information provided. Following the suggested problem-solving routine will help you arrive at a solution.

EXAMPLE 13.8 Is the crown made of gold?

You need to determine if a crown is made from pure gold or some less valuable metal. From Table 13.1 you know that the density of gold is $19{,}300\ \text{kg/m}^3$. You find that the force that a string attached to a spring scale exerts on the crown is 25.0 N when the crown hangs in air and 22.6 N when the crown hangs completely submerged in water.

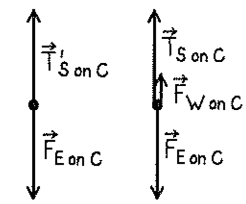
You can determine the mass of the crown easily from the measurement of the scale when the crown hangs in air. But how can you determine the volume from the given information? Crowns have irregular shapes, and it would be difficult to determine its volume by simple measurements and calculations.

Sketch and translate We draw a sketch of the situation and label the givens. If you could measure the mass m_{C} of the crown and its volume V_{C}, you could calculate the density $\rho_{\text{C}} = m_{\text{C}}/V_{\text{C}}$ of the crown—it should be $19{,}300\ \text{kg/m}^3$.



Simplify and diagram Let's follow the recommended strategy and see what happens. First, we draw force diagrams for the crown hanging in air and again when hanging in water. When the crown is in air, the upward force exerted by the string attached to the spring scale balances the downward force exerted by Earth. We ignore the buoyant force that air exerts on the crown when hanging in air, since it will be very small in magnitude compared with the other forces exerted on the crown.

When the crown is in water, the upward force exerted by the string (the force measured by the scale) and the upward buoyant force that the water exerts on the crown combine to balance the downward gravitational force that Earth exerts on the crown.



Represent mathematically Since the crown is in equilibrium, the forces exerted on it must add to zero in both cases. When the crown is hanging in air, the vertical component form of Newton's second law is

$$0 = \Sigma F_y = T'_{S\ on\ C} + (-F_{E\ on\ C})$$

where $T'_{S\ on\ C}$ is the 25.0-N string tension force exerted on the crown when it is suspended in air. Thus,

$$F_{E\ on\ C} = m_C g = 25.0\ N$$

or

$$m_C = \frac{25.0\ N}{9.8\ N/kg} = 2.55\ kg$$

The vertical component form of Newton's second law when the crown hangs in water becomes (the upward direction is positive):

$$\Sigma F_y = T_{S\ on\ C} + F_{W\ on\ C} + (-F_{E\ on\ C}) = 0$$

where $T_{S\ on\ C} = 22.6$ N is the magnitude of the string tension force, and the buoyant force that the water exerts on the crown is $F_{W\ on\ C} = \rho_W V_C g$. Substituting in the above, we get

$$T_{S\ on\ C} + \rho_W V_C g - m_C g = 0$$

Solve and evaluate We see now that the last equation can be used to determine the volume of the crown:

$$V_C = \frac{m_C g - T_{S\ on\ C}}{\rho_W g} = \frac{25.0\ N - 22.6\ N}{(1000\ kg/m^3)(9.8\ N/kg)} = 0.000245\ m^3$$

We now know the crown mass and volume and can calculate its density:

$$\rho = \frac{m}{V} = \frac{2.55\ kg}{0.000245\ m^3} = 10{,}400\ kg/m^3$$

Oops! Since $10{,}400\ kg/m^3$ is much less than the $19{,}300\ kg/m^3$ density of gold, the crown is not made of pure gold. The goldsmith must have combined the gold with some less expensive metal.

Try it yourself What is the density of the crown if the scale reads 0 when submerged in water?

Answer $1000\ kg/m^3$.

REVIEW QUESTION 13.6 Two objects have the same volume, but one is heavier than the other. When they are completely submerged in oil, on which one does the oil exert a greater buoyant force?

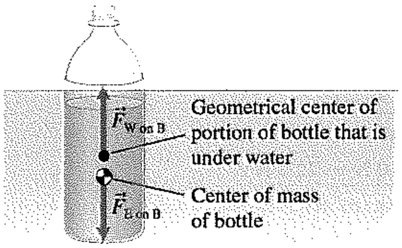
13.7 Ships, balloons, climbing, and diving

As we learned in Section 13.1, whether an object floats or sinks depends on its density relative to the density of the fluid. The reason for this lies in the interactions of the object with the fluid and Earth. Specifically, the magnitude of the buoyant force is $F_{F\ on\ O} = \rho_{fluid} V_{subm}\, g$, and the magnitude of the force exerted by Earth is $F_{E\ on\ O} = \rho_{object} V_{object}\, g$. These forces are exerted in the opposite directions. The relative densities of the fluid and the object and consequently the relative magnitudes of the forces determine what happens to the object when placed in the fluid.

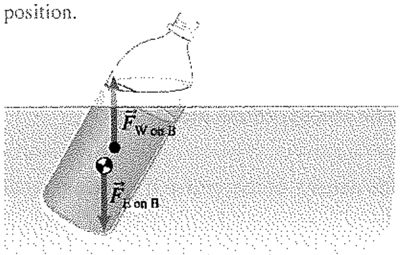
- If the object's density is less than that of the fluid $\rho_{object} < \rho_{fluid}$, then $\rho_{object} V_{object}\, g < \rho_{fluid} V_{object}\, g$; the object floats *partially* submerged since the buoyant force can balance the gravitational force with less than the entire object below the surface of the fluid.
- If the densities are the same $\rho_{object} = \rho_{fluid}$, then $\rho_{object} V_{object}\, g = \rho_{fluid} V_{object}\, g$; the sum of the forces exerted on the object is zero and it remains wherever it is placed totally submerged at any depth in the fluid.
- If the object is denser than the fluid $\rho_{object} > \rho_{fluid}$, then $\rho_{object} V_{object}\, g > \rho_{fluid} V_{object}\, g$; the magnitude of the gravitational force is always greater than the magnitude of the buoyant force. The object sinks until it reaches the bottom of the container.

FIGURE 13.14 Making a bottle float with stable equilibrium.

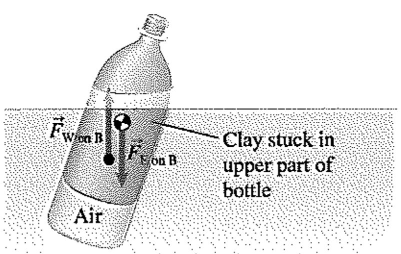
(a) Bottle partially filled with sand floats upright if the center of mass is below the geometrical center of bottle.



(b) If the bottle is tipped, torques due to forces exerted on the bottle return it to the upright position.



(c) If this bottle tips slightly, torques due to forces exerted on the bottle cause it to overturn.



These cases show that by changing the average density of an object relative to the density of the fluid, the object can be made to float or sink in the same fluid. In this section we investigate this phenomenon and its many practical applications.

Building a stable ship

For years ships were made of wood. In the middle of the 17th century, people decided to try building metal ships. Many thought that this idea was absurd: iron is denser than water and an iron boat would certainly sink. In 1787 British engineer John Wilkinson succeeded in building the first iron ship that did not sink. Since the middle of the 19th century, large ships have been made primarily of steel, which is less dense than iron but much denser than water. These ships can float because part of the volume of a ship is filled with air, which reduces the average density of the ship to a density lower than that of water.

Making a ship or a raft float is only part of the challenge of building watercraft. Another problem is to maintain stable equilibrium for the ship, allowing it to right itself if it tilts to one side due to wind or rough seas. Refresh your knowledge of stable equilibrium (Section 8.6) before you read on.

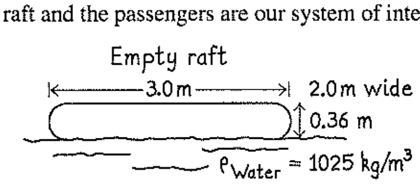
Consider a floating bottle partially filled with sand. Earth exerts a gravitational force at the center of mass of the bottle (Figure 13.14a). The buoyant force exerted by the water on the bottle is effectively exerted at the geometrical center of the part of the bottle that is underwater, which equals the center of mass of the displaced water. If this point is above the center of mass of the bottle, then any slight tipping causes these forces to produce a torque that attempts to return the bottle to an upright position (see Figure 13.14b). However, if the geometrical center of the part of the bottle that is underwater is below the center of mass of the bottle, slight tipping causes the gravitational force to produce a torque that enhances the tipping—unstable equilibrium (Figure 13.14c).

Although ships are more complicated than water bottles filled with sand, it is important to load a ship in such a way that when it begins to heel and one side of the hull begins to rise from the water, the center of mass of the displaced water is above the center of mass of the ship. This is why ships have their cargo stored at the bottom.

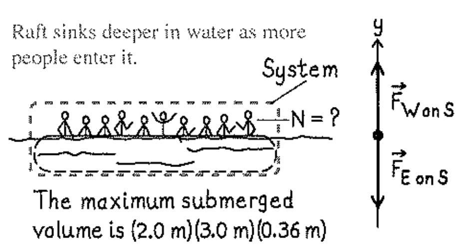
EXAMPLE 13.9 **Should we take this trip?**

The top of an empty life raft of cross-sectional area 2.0 m × 3.0 m is 0.36 m above the waterline. How many 75-kg passengers can the raft hold before water starts to flow over its top? The raft is in seawater of density 1025 kg/m^3.

Sketch and translate We make a sketch of the unloaded raft. As people get on the raft, it sinks deeper into the water, and the upward buoyant force increases until the raft reaches a maximum submerged volume, when the maximum number of people are on board. The maximum submerged volume is $V_{\text{subm}} = 2.0\ \text{m} \times 3.0\ \text{m} \times 0.36\ \text{m} = 2.16\ \text{m}^3$. We need to determine the maximum buoyant force the seawater can exert on the raft and then decide how to convert this into the number of passengers the raft can hold. The raft and the passengers are our system of interest.



Simplify and diagram We have no information about the mass of the raft; thus we will assume it is negligible. We draw a sketch of the filled raft and a force diagram for the raft with passengers (the system). The vertical axis points up. There are two forces exerted on the system: the upward force exerted by the water $\vec{F}_{\text{W on S}}$ of magnitude $\rho_{\text{water}} g V_{\text{subm}}$ and the downward force exerted by Earth $\vec{F}_{\text{E on S}}$. The magnitude of the force Earth exerts on N people is $N_{\text{people}} m_{\text{person}} g$. As the system is in equilibrium, the net force exerted on it is zero.



Represent mathematically Using the upward direction as positive, apply the vertical component form of Newton's second law:

$$\Sigma F_y = F_{\text{W on S}} + (-F_{\text{E on S}}) = 0$$

$$\rho_{\text{water}} g V_{\text{subm}} - N_{\text{people}} m_{\text{person}} g = 0$$

Assuming that all people have the same mass, we find the number of people:

$$N_{\text{people}} = \frac{\rho_{\text{water}} V_{\text{subm}} g}{m_{\text{person}} g} = \frac{\rho_{\text{water}} V_{\text{subm}}}{m_{\text{person}}}$$

Solve and evaluate

$$N_{\text{people}} = \frac{\rho_{\text{water}} V_{\text{subm}}}{m_{\text{person}}}$$

$$= \frac{(1025\ \text{kg/m}^3)(0.36\ \text{m} \times 2.0\ \text{m} \times 3.0\ \text{m})}{75\ \text{kg}} = 29.5$$

The raft can precariously hold 29 passengers, which is a reasonable number. The number is inversely proportional to the mass of a person. This makes sense—the heavier the people, the fewer of them the raft should hold. The units, dimensionless, also make sense. We assumed that the raft has negligible mass. If we take the mass into account, the number of people will be smaller.

Try it yourself Suppose that 10 people of average mass 80 kg entered the raft. Now how far would the water line be from the top of the raft?

Answer The raft would sink 0.13 m into the water, and the water line would be 0.23 m below the top of the raft.

Ballooning

Balloons used for transportation are filled with hot air. Why hot air? The density of 100 °C air is 0.73 times the density of 0 °C air. Thus, balloonists can adjust the average density of the balloon (the balloon's material, people, equipment, etc.) to match the density of air so that the balloon can float at any location in the atmosphere (up to certain limits). A burner under the opening of the balloon regulates the temperature of the air inside the balloon and hence its volume and density. This allows control over the buoyant force that the outside cold air exerts on the balloon. The same approach that we used in Example 13.9 allows us to predict that a balloon with radius of 5.0 m and a mass of 20.0 kg filled with air at the temperature of 100 °C can carry about 160 kg (three slim 53-kg people or two medium-mass 80-kg people). This is not a heavy load.

At one time, hydrogen was used in closed balloons instead of air. The density of hydrogen is 1/14 times the density of air. Unfortunately, hydrogen can burn explosively in the presence of oxygen. The hydrogen-filled Hindenburg, a German airship, caught fire and exploded in 1937, killing 36 people (see Figure 13.15). Balloonists then turned to helium—an inert gas that does not interact readily with other types of atoms.

FIGURE 13.15 The Hindenburg explosion.

Effects of altitude on humans

Table 13.5 on the next page shows that atmospheric pressure decreases with altitude. Therefore, climbers and balloonists have to guard against altitude sickness, caused by the low pressure and lack of oxygen. Below 3000 m, altitude has little effect on performance. Between 3000 m and 4600 m, climbers experience compensated hypoxia—increased heart and breathing rates. Between 4600 m and 6000 m, manifest hypoxia sets in. Heart and breathing rates increase dramatically, and cognitive and sensory function and muscle control decline. Climbers may feel lethargy and euphoria and even experience hallucinations. Between 6000 m and 8000 m, climbers undergo critical hypoxia, characterized by rapid loss of muscular control, loss of consciousness, and possibly death.

These symptoms were exhibited clearly on April 15, 1875 by three French balloon pioneers attempting to set an altitude record. They carried bags of oxygen with them, but as their elevation increased, slowly lost the mental awareness needed to use the bags. Instruments indicate that the balloon reached a maximum elevation of 8600 m twice. During the second time, two of the balloonists died. The third lost consciousness but survived.

TABLE 13.5 Pressure of air and pressure due to oxygen in the air (called partial pressure) at different elevations

Location	Elevation (m)	P_{air} (atm)	P_{oxygen} (atm)
Sea level	0	1.0	0.21
Mount Washington	1917	0.93	0.18
Pikes Peak	4301	0.59	0.12
Mount McKinley	6190	0.47	0.10
Mount Everest	8848	0.34	0.07
Jet travel	12,000	0.23	0.05

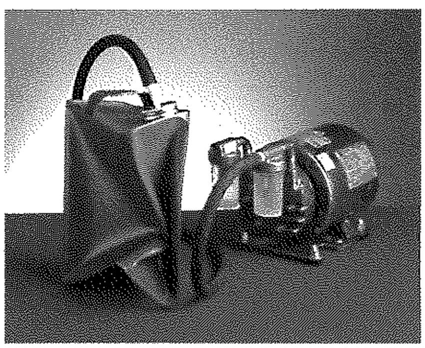
Scuba diving

The sport of scuba diving depends on an understanding of fluid pressure and buoyant force to avoid cases of excess internal pressure, oxygen overload, and decompression sickness.

Assuming that the surface area of your body is about 2 m^2, the air exerts a 200,000-N force (about 20 tons) on the surface of your body. Fortunately, fluids inside the body push outward and balance the force exerted by the outside air. For example, the pressure inside your lungs is approximately atmospheric pressure. What would happen if the fluid pressure on the inside remained constant while the pressure on the outside doubled or tripled? Would you be crushed, the way a can or barrel is crushed by outside air pressure when the air pressure inside the can is much lower than the pressure outside (Figure 13.16)? Scuba divers face this problem.

FIGURE 13.16 External air pressure collapses an evacuated can.

When air is pumped out of a can, outside air pressure causes it to collapse.



We know that atmospheric pressure (1 atm) is equivalent to the pressure of a 10-m column of water. Therefore, at depth $d = 10$ m, the water pressure is 2 atm. At 40 m below the water surface, the pressure is about 5 atm. This would surely be a problem for a scuba diver if the internal pressure were only 1 atm!

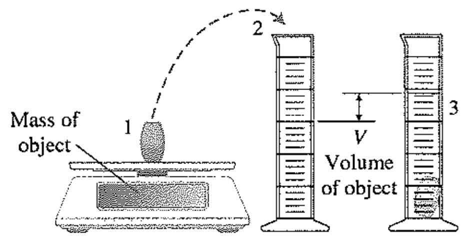
To avoid this problem, divers breathe compressed air. While moving slowly downward, a diver adjusts the pressure outlet from the compressed air tank in order to accumulate gas from the cylinder into her lungs and subsequently into other body parts, increasing the internal pressure to balance the increasing external pressure. If a diver returns to the surface too quickly, the great gas pressure in the lungs can force bubbles of gas into the bloodstream. These bubbles can behave like blood clots, blocking blood flow to the brain and possibly causing death. Blood vessels can rupture if the pressure difference between the inside and outside of the vessel is too great. Thus, a diver rises to the surface slowly so that pressure changes gradually and bubbles of gas do not form. This gradual process is called *decompression*.

When humans travel to dangerous environments (mountaintops, the deep sea, or outer space), physics intersects with human physiology. A careful understanding of gases, liquids, and the effects of changing pressures on the human body is needed to allow humans to survive in these places. As we explore the universe, we will need to learn to adapt to ever more challenging environments.

REVIEW QUESTION 13.7 A ship's waterline marks the maximum safe depth of the ship in the water when it has a full cargo. An empty ship is at the dock with its waterline somewhat above the water level. How could you estimate its maximum cargo?

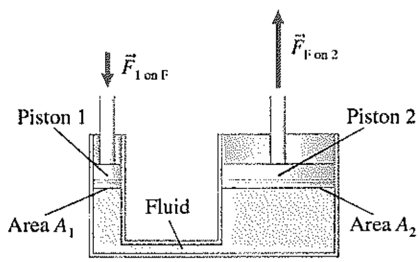
Summary

Density ρ The ratio of the mass m of a substance divided by the volume of that substance. (Section 13.1)



$$\rho = \frac{m}{V}$$ Eq. (13.1)

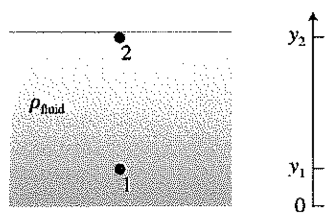
Pascal's first law—hydraulic lift An increase in the pressure in one part of an enclosed fluid increases the pressure throughout the fluid. In a hydraulic lift, a small force F_1 exerted on a small piston of area A_1 can cause a large force F_2 to be exerted on a large piston of area A_2. (Section 13.2)



For a hydraulic lift:

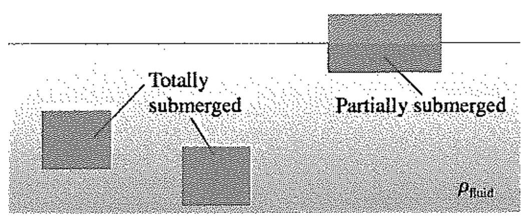
$$F_{\text{F on 2}} = PA_2 = (A_2/A_1)\,F_{\text{1 on F}}$$ Eq. (13.2)

Pascal's second law—variation of pressure with depth On a vertical upward-pointing y-axis, the pressure of a fluid P_1 at position y_1 depends on the pressure P_2 at position y_2 and on the density of the fluid. (Section 13.3)



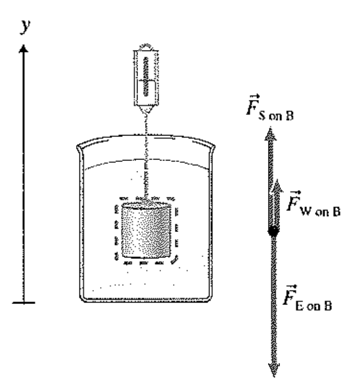
$$P_1 = P_2 + \rho_{\text{fluid}}(y_2 - y_1)g$$ Eq. (13.3)

Buoyant force A fluid exerts an upward-pointing buoyant force on an object totally or partially immersed in the fluid. The force depends on the density of the fluid and on the volume of the fluid displaced. Once the object is totally submerged, the buoyant force does not change. (Section 13.5)



$$F_{\text{F on O}} = \rho_{\text{fluid}} V_{\text{displaced}}\, g$$ Eq. (13.5)

Newton's second law Use the standard problem-solving strategies with this law (sketches, force diagrams, math descriptions) to find some unknown quantity. The problems often involve the buoyant force, pressure, and density. (Section 13.6)



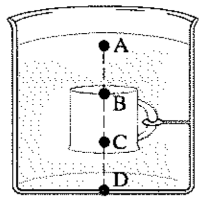
$$a_y = \frac{\Sigma F_y}{m}$$ Eq. (3.7y)

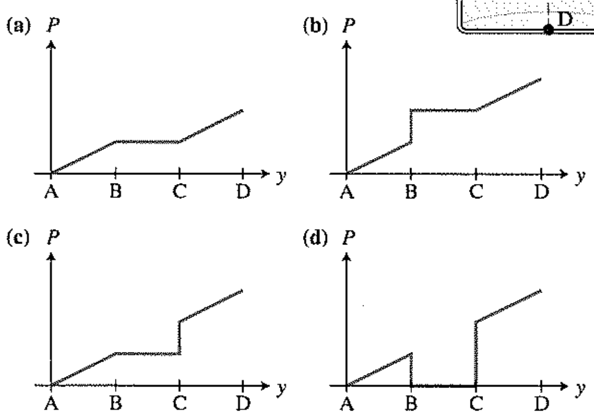
Questions

Multiple Choice Questions

1. Rank in increasing order the pressure that the *italicized objects* exert on the surface.
   I. A person standing with *bare feet* on the floor
   II. A person in *skis* standing on snow
   III. A person in *Rollerblades* standing on a road
   IV. A person in *ice skates* standing on ice
   (a) I, II, III, IV (b) IV, III, I, II (c) IV, III, II, I
   (d) III, II, IV, I (e) II, I, III, IV
2. Choose a device that reduces the pressure caused by a force.
   (a) Scissors (b) Knife (c) Snowshoes
   (d) Nail (e) Syringe
3. What does it mean if the density of a material equals 2000 kg/m^3?
   (a) The mass of the material is 2000 kg.
   (b) The volume of the material is 1 m^3.
   (c) The ratio of the mass of any amount of this material to the volume is equal to 2000 kg/m^3.
4. An upside-down mug with some air trapped in it is fixed under water, as shown in Figure Q13.4. Which qualitative pressure-versus-position graph correctly shows how the gauge pressure changes along the dashed line through the mug from A to D?

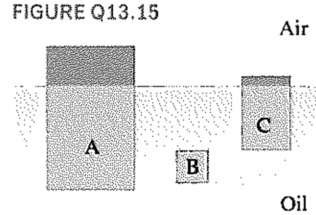
FIGURE Q13.4





5. If you hold a cylinder vertically, what is the net force exerted by the atmospheric pressure on it?
   (a) Downward (b) Upward (c) Zero
6. How do we know that a fluid exerts an upward force on an object submerged in the fluid?
   (a) Fluid pushes on the object in all directions.
   (b) The reading of a scale supporting the object when submerged in the fluid is less than when not in the fluid.
   (c) The fluid pressure on the bottom of the object is greater than the pressure on the top.
   (d) Both b and c are correct.
7. When you suspend an object from a spring scale, it reads 15 N. Then you place the same object and scale under a vacuum jar and pump out the air. What happens to the reading of the scale?
   (a) It increases slightly.
   (b) It decreases slightly.
   (c) It says the same.
   (d) Don't have enough information to answer
8. Why can't a suction pump lift water higher than 10.3 m?
   (a) Because it does not have the strength to pull up higher
   (b) Because the atmospheric pressure is equal to the pressure created by a 10.3-m-high column of water
   (c) Because suction pumps are outdated lifting devices
   (d) Because most suction cups have an opening to the bulb that is too narrow
9. If Torricelli had a wider tube in his mercury barometer, what would the height of the mercury column in the tube do?
   (a) Decrease (b) Increase (c) Stay the same
10. A wooden cube is floating in a fish tank that is filled with water. Imagine that you take this setup to a space station on the Moon. Air pressure and temperature inside the station are similar to conditions on Earth. After bringing the setup from Earth to the space station, you will observe that (choose all correct statements)
    (a) the amount of water displaced by the cube increases.
    (b) the amount of water displaced by the cube decreases.
    (c) the amount of water displaced by the cube stays the same.
    (d) the buoyant force exerted on the cube increases.
    (e) the buoyant force exerted on the cube decreases.
    (f) the buoyant force exerted on the cube stays the same.
    (g) the pressure at the bottom of the fish tank is about 1/6 of the value on Earth.
    (h) the pressure at the bottom of the fish tank is significantly less than 1/6 of the value on Earth.
    (i) the pressure at the bottom of the fish tank is slightly less than the value on Earth.
    (j) the pressure at the bottom of the fish tank is the same as on Earth.
11. Two identical beakers with the same amount of water sit on the arms of an equal arm balance. A wooden block floats in one of them. What does the scale indicate?
    (a) The beaker with the block is heavier.
    (b) The beaker without the block is heavier.
    (c) The beakers weigh the same.
12. A piece of steel and a bag of feathers are suspended from two spring scales in a vacuum. Each scale reads 100 N. What happens when you repeat the experiment outside under normal conditions?
    (a) The scale with feathers reads more than the scale with steel.
    (b) The scale with feathers reads less than the scale with steel.
    (c) The scales have the same reading, but the reading is less than the reading in a vacuum.
13. A metal boat floats in a pool. What happens to the level of the water in the pool if the boat sinks?
    (a) It rises.
    (b) It falls.
    (c) It stays the same.
14. When a boat sails from seawater to fresh water, the buoyant force exerted on the boat
    (a) decreases. (b) increases. (c) stays the same.
15. Three blocks are floating in oil as shown in Figure Q13.15. Which block has the highest density?
    (a) A (b) B (c) C
    (d) All blocks have the same density.
16. Three blocks are floating in oil as shown in Figure Q13.15. On which block does the oil exert the greatest buoyant force?
    (a) A (b) B (c) C
    (d) The oil exerts the same force on all of them.

FIGURE Q13.15

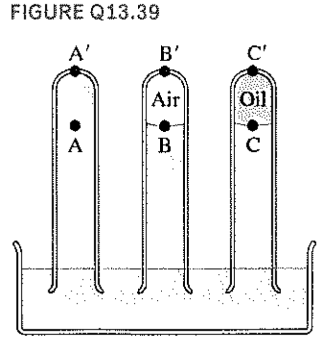


Conceptual Questions

17. Describe a method to measure the density of a liquid.
18. How can you determine the density of air?
19. Design an experiment to determine whether air has mass.
20. Does air exert a net upward force or a net downward force on an object submerged in the air? How can you test your answer experimentally?
21. What causes the pressure that air exerts on a surface that is in the air?
22. Why, when you fill a teapot with water, is the water always at the same level in the teapot and in the spout?
23. What experimental evidence supports Pascal's first law?

24. Fill a plastic cup to the very top with water. Put a piece of paper on top of the cup so that the paper covers the cup at the edges and is not much bigger than the surface of the cup. Turn the cup and paper upside down (practice over the sink first) and hold the bottom of the cup (now on the top). Why doesn't the water fall out of the cup?
25. Why does a fluid exert a net upward force on an object submerged in the fluid?
26. Describe how you could predict whether an object will float or sink in a particular liquid without putting it into the liquid.
27. Why can you lift objects while in water that are too heavy to lift when in the air?
28. When placed in a lake, a solid object either floats on the surface or sinks. It does not float at some intermediate location between the surface and the bottom of the lake. However, a weather balloon floats at some intermediate distance between Earth's surface and the top of its atmosphere. Explain.
29. A flat piece of aluminum foil sinks when placed under water. Take the same piece and shape it so that it floats in the water. Explain why the method worked.
30. Ice floats in water in a beaker. Will the level of the water in the beaker change when the ice melts? Explain.
31. The density of ice at 0 °C is less than the density of water at 0 °C. How is this related to the existence of life on Earth?
32. How would you determine the density of an irregular-shaped unknown object if (a) it sinks in water and (b) it floats in water? List all the steps and explain the reasoning behind them.
33. Why do people sink in fresh water and in most seawater (if they do not make an effort to stay afloat) but do not sink in the Dead Sea?
34. A bucket filled to the top with water has a piece of ice floating in it. Will the pressure on the bottom of the bucket change when the ice melts? Explain.
35. Marjory thinks that the mass of a fluid above a certain level should affect the pressure at this level. Describe how you will test her idea.
36. You are holding a brick that is completely submerged in water. Draw a force diagram for the brick. Why does it feel lighter in water than when you hold it in the air?
37. A bucket filled with water has a piece of ice floating in it. Will the level of water rise when the ice melts? Justify your answer.
38. Explain qualitatively and quantitatively how we drink through a straw. Make sure you can account for the water going up the length of the straw.
39. Three test tubes are inverted in a Petri dish as shown in Figure Q13.39. The first is completely filled with water, the second has an air bubble at the top, and the third has oil at the same level as the air bubble. (a) Rank the pressures at points A, B, and C from largest to smallest. (b) Rank the pressures at points A′, B′, and C′ from largest to smallest.

FIGURE Q13.39



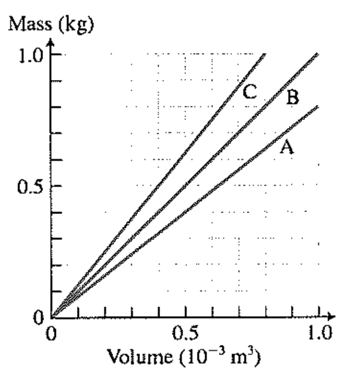
Problems

Below, BIO indicates a problem with a biological or medical focus. Problems labeled EST ask you to estimate the answer to a quantitative problem rather than derive a specific answer. Asterisks indicate the level of difficulty of the problem. Problems with no * are considered to be the least difficult. A single * marks moderately difficult problems. Two ** indicate more difficult problems.

13.1 Density

1. Determine the average density of Earth. What data did you use? What assumptions did you make?
2. * EST **Height of atmosphere** Use data for the normal pressure and the density of air near Earth's surface to estimate the height of the atmosphere, assuming it has *uniform* density. Indicate any additional assumptions you made. Are you on the low or high side of the real number?
3. EST A single-level home has a floor area of 200 m^2 with ceilings that are 2.6 m high. Estimate the mass of the air in the house.
4. * BIO A diet decreases a person's mass by 5%. Exercise creates muscle and reduces fat, thus increasing the person's density by 2%. Determine the percent change in the person's volume.
5. **Pulsar density** A pulsar, an extremely dense rotating star made of neutrons, has a density of 10^{18} kg/m^3. Determine the mass of a pulsar contained in a volume the size of your fist (about 200 cm^3).
6. Use the graph lines in Figure P13.6 to determine the densities of the three liquids, A, B, and C, in SI units. If you place them in one container, how will they position themselves? How does the density of each liquid change as its volume increases? As its mass decreases? Compare the masses of the three liquids when they occupy the same volume. Compare the volumes of the three liquids when they have the same mass.

FIGURE P13.6



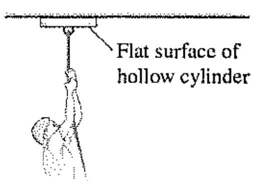
7. Imagine that you have gelatin cut into three cubes: the side of cube A is *a* cm long, the side of cube B is double the side of A, and the side of cube C is three times the side of A. Compare the following properties of the cubes: (a) density, (b) volume, (c) surface area, (d) cross-sectional area, and (e) mass.
8. An object made of material A has a mass of 90 kg and a volume of 0.45 m^3. If you cut the object in half, what would be the density of each half? If you cut the object into three pieces, what would be the density of each piece? What assumptions did you make?
9. You have a steel ball that has a mass of 6.0 kg and a volume of 3.0×10^{-3} m^3. How can this be?
10. * A material is made of molecules of mass 2.0×10^{-26} kg. There are 2.3×10^{29} of these molecules in a 2.0-m^3 volume. What is the density of the material?
11. You compress all the molecules described in Problem 13.10 into 1.0 m^3. Now what is the density of the material? What type of material could possibly behave this way?
12. * Bowling balls are heavy. However, some bowling balls float in water. Use available resources to find the dimensions of a bowling ball and explain why some balls float while others do not.
13. * EST Estimate the average density of a glass full of water and then the glass when the water is poured out (do not forget the air that now fills the glass instead of water).

13.2 Pressure inside a fluid

14. * Anita holds her physics textbook and complains that it is too heavy. Andrew says that her hand should exert no force on the book because the atmosphere pushes up on it and balances the downward pull of Earth on the book (the book's weight). Jim disagrees. He says that the atmosphere presses down on things and that is why they feel heavy. Who is correct? Approximately how large is the force that the atmosphere exerts on the bottom of the book? Why does this force not balance the force exerted by Earth on the book?

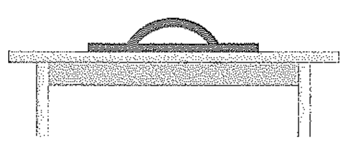
15. * The air pressure in the tires of a 980-kg car is $3.0 \times 10^5\ N/m^2$. Determine the average area of contact of each tire with the road. Indicate any assumptions that you made.
16. * EST Estimate the pressure that you exert on the floor while wearing hiking boots. Now estimate the pressure under each heel if you change into high-heeled shoes. Indicate any assumptions you made.
17. **Hydraulic car lift** You are designing a hydraulic lift for a machine shop. The average mass of a car it needs to lift is about 1500 kg. What should be the specifications on the dimension of the pistons if you wish to exert a force on a smaller piston of not more than 500 N? How far down will you need to push the piston in order to lift the car 30 cm?
18. EST **Force of air on forehead** Estimate the force that air exerts on your forehead. Describe the assumptions you made.
19. * EST A 30-cm-diameter cylindrical iron plunger is held against the ceiling, and the air is pumped from inside it. A 72-kg person hangs by a rope from the plunger (Figure P13.19). List the quantities that you can estimate about the situation and estimate them. Make assumptions if necessary.

FIGURE P13.19



20. You have a rubber pad with a handle attached to it (Figure P13.20). If you press the pad firmly on a smooth table, it is impossible to lift it off the table. Why? What force would you need to exert on the handle to lift it? The surface area of the pad is $0.023\ m^2$.

FIGURE P13.20

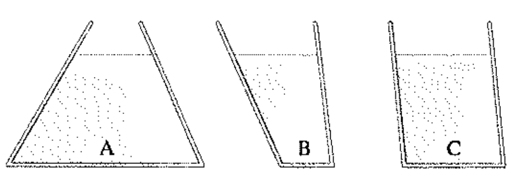


21. * EST **Toy bow and arrow** A child's toy arrow has a suction cup on one end. When the arrow hits the wall, it sticks. Draw a force diagram for the arrow stuck on the wall and estimate the magnitudes of the forces exerted on it when it is in equilibrium. The mass of the arrow is about 10 g. Why are the words "suction cup" not appropriate?

13.3 Pressure variation with depth

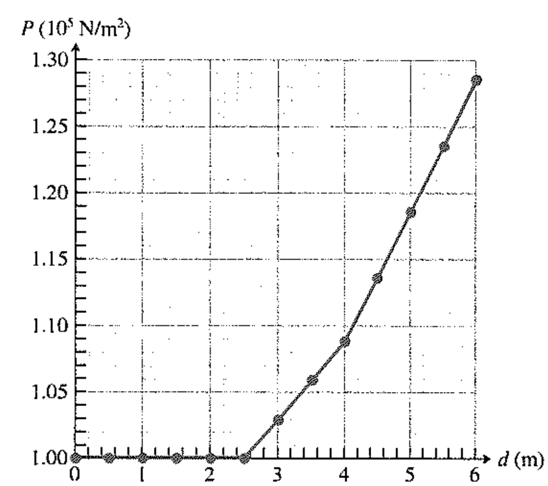
22. * **Pressure on the Titanic** The Titanic rests 4 km (2.5 miles) below the surface of the ocean. What physical quantities can you determine using this information?
23. You have three reservoirs (Figure P13.23). Rank the pressures at the bottom of each and explain your rankings. Then rank the net force that the water exerts on the bottom of each reservoir. Explain your rankings.

FIGURE P13.23



24. **Water reservoir and faucet** The pressure at the top of the water in a city's gravity-fed reservoir is $1.0 \times 10^5\ N/m^2$. Determine the pressure at the faucet of a home 42 m below the reservoir.
25. **Dutch boy saves Holland** An old story tells of a Dutch boy who used his fist to plug a 2.0-cm-diameter hole in a dike that was 3.0 m below sea level, thus preventing the flooding of part of Holland. What physical quantities can you determine from this information? Determine them.
26. BIO EST **Blood pressure** Estimate the pressure of the blood in your brain and in your feet when standing, relative to the average pressure of the blood in your heart of $1.3 \times 10^4\ N/m^2$ above atmospheric pressure.
27. * BIO **Intravenous feeding** A glucose solution of density $1050\ kg/m^3$ is transferred from a collapsible bag through a tube and syringe into the vein of a person's arm. The blood pressure in the arm exceeds the atmospheric pressure by $1400\ N/m^2$. How high above the arm must the top of the liquid in the bottle be so that the pressure in the glucose solution at the needle exceeds the pressure of the blood in the arm?
28. * **Mountain climbing** Determine the change in air pressure as you climb from elevation of 1650 m at the timberline of Mount Rainier to its 4392-m summit, assuming an average air density of $0.82\ kg/m^3$. Will the real change be more or less than the one you calculated? Explain.
29. BIO EST **Giraffe raises head** Estimate the pressure change of the blood in the brain of a giraffe when it lifts its head from the grass to eat a leaf on an overhead tree. Without special valves in its circulatory system, the giraffe could easily faint when lifting its head.
30. * A truck transporting chemicals has crashed, and some dangerous liquid has spilled onto the ground and possibly entered a water well. An inspector fixes a pressure sensor to the end of a long string and lets the sensor slowly descend from the top of the well to the bottom. Using this device, he obtains the graph in Figure P13.30 that shows how the pressure P in the well changes with distance d measured from the top of the well. (a) Explain what features of the graph support the idea that there is another liquid in the well in addition to water. (b) Determine the density of the unknown liquid. Is the liquid above the water or below the water? (c) Determine the depth of the water, the depth of the unknown liquid, and the depth of the well.

FIGURE P13.30

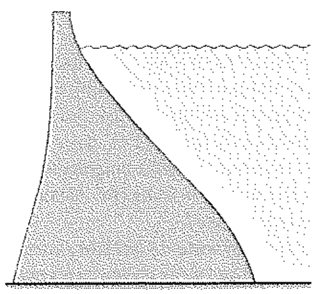


31. **Drinking through a straw** You are drinking water through a straw in an open glass. Select a small volume of water in the straw as a system and draw a force diagram for the water inside this volume that explains why the water goes up the straw.
32. * **More straw drinking** While you are drinking through the straw, the pressure in your mouth is 30 mm Hg below atmospheric pressure. What is the maximum length of a straw in an open glass that you can use to drink a fruit drink of density $1200\ kg/m^3$?
33. * Your office has a $0.020\ m^3$ cylindrical container of drinking water. The radius of the container is about 14 cm. When the container is full, what is the gauge pressure that the water exerts on the sides of the container halfway down from the top? All the way down?
34. * BIO EST **Eardrum** Estimate the net force on your $0.5\text{-}cm^2$ eardrum that air exerts on the inside and the outside after you drive from Denver, Colorado (elevation 1609 m) to the top of Pikes Peak (elevation 4301 m). Assume that the air pressure inside and out are balanced when you leave Denver and that the average density of the air is $0.80\ kg/m^3$. What other assumptions did you make?
35. BIO **Eardrum again** You now go snorkeling. What is the net force exerted on your eardrum when you are 2.4 m under the water, assuming the pressure was equalized before the dive?

36. Water and oil are poured into opposite sides of an open U-shaped tube. The oil and water meet at the exact center of the U at the bottom of the tube. If the column of oil of density 900 kg/m^3 is 16 cm high on one side, how high is the water on the other side?

37. * Examine the vertical cross section of the Hoover Dam shown in Figure P13.37. Explain why the dam is thicker at the bottom than at the top.

FIGURE P13.37

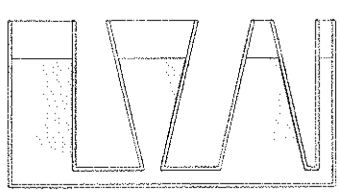


38. * A test tube of length L and cross-sectional area A is submerged in water with the open end down so that the edge of the tube is a distance h below the surface. The water goes up into the tube so its height inside the tube is l. Describe how you can use this information to decide whether the air that was initially in the tube obeys the mathematical description for an isothermal process (Boyle's law). List your assumptions.

39. Half of a 20-cm-tall beaker is filled with glycerol (density 1259 kg/m^3) and the other half with olive oil (density 800 kg/m^3). (a) Draw a graph that shows how the density of the liquids in the beaker changes with the distance from the surface to the bottom of beaker. (b) Draw a graph that shows how the pressure in the liquids changes with the distance from the surface to the bottom of the beaker.

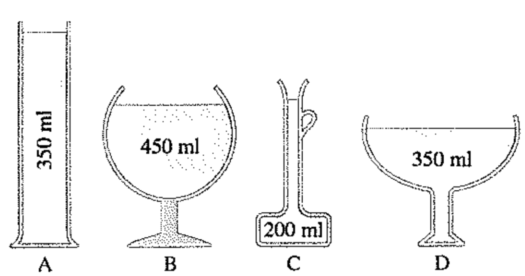
40. Blaise Pascal found a seemingly paradoxical situation when he poured water into the apparatus shown in Figure P13.40. The water level was the same in all four parts of the apparatus despite differences in the shapes of the parts and the masses of water in each part. Explain qualitatively the outcome of Pascal's experiment.

FIGURE P13.40



41. Four containers are filled with different volumes of water as shown in Figure P13.41. Rank the containers in order of decreasing pressure that the water exerts on the bottom of the containers.

FIGURE P13.41



42. **Venus pressure and underwater pressure** Atmospheric pressure on Venus is $9.0 \times 10^6\ N/m^2$. How deep underwater on Earth would you have to go to feel the same pressure?

13.4 Measuring atmospheric pressure

43. The reading of a barometer in your room is 780 mm Hg. What does this mean? What is the pressure in pascals?

44. How long would Torricelli's barometer have had to be if he had used oil of density 950 kg/m^3 instead of mercury?

45. Sometimes gas pressure is measured with a device called a *liquid manometer* (Figure P13.45). Explain how this instrument can be used to measure the pressure of gas in a bulb attached to one of the tubes.

FIGURE P13.45

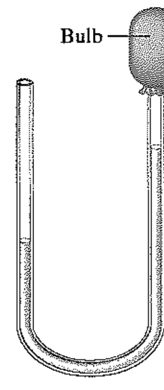
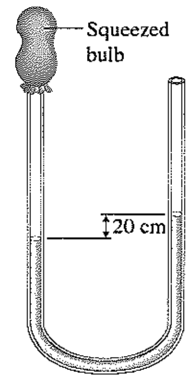


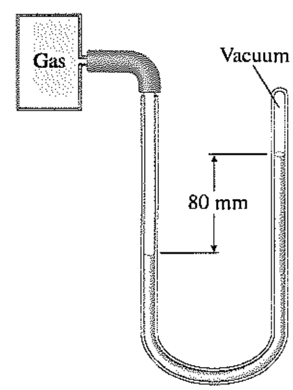
FIGURE P13.46



46. You use a liquid manometer with water to measure the pressure inside a rubber bulb. Before you squeeze the bulb, the water is at the same level in both legs of the tube. After you squeeze the bulb, the water in the opposite leg rises 20 cm with respect to the leg connected to the bulb (Figure P13.46). What is the pressure in the bulb? What assumptions did you make? How will the answer change if the assumptions are not valid?

47. * In a mercury-filled manometer (Figure P13.47), the open end is inserted into a container of gas and the closed end of the tube is evacuated. The difference in the height of the mercury is 80 mm. The radius of the connecting tube is 0.50 cm. (a) Determine the pressure inside the container in newtons per square meter. (b) An identical manometer has a connecting tube that is twice as wide. If the difference in the height of the mercury is the same, then what is the pressure in the container?

FIGURE P13.47



48. Examine the reading of the manometer that you use to measure the pressure inside car tires. What are the units? Does the manometer measure the absolute pressure of the air inside the tires or gauge pressure? How do you know?

13.5 Buoyant force

49. Draw a force diagram for an object that is floating at the surface of a liquid. Is the force exerted by air on the object included in your diagram? Explain.

50. Draw a cubic object that is completely submerged in a fluid but not resting on the bottom of the container. Then draw arrows to represent the forces exerted by the fluid on the top, sides, and bottom of the object. Make the arrows the correct relative lengths. What is the direction of the total force exerted by the fluid on the object?

51. Three people are holding three identical sealed metal containers, ready to release them. Container A is filled with air at atmospheric pressure and room temperature, container B is filled with helium at atmospheric pressure and room temperature, and container C is evacuated (all air has been pumped out). Draw force diagrams for the containers after they are released.

52. * Four cubes of the same volume are made of different materials: lead (density 11,300 kg/m^3), aluminum (density 2700 kg/m^3), wood (density 800 kg/m^3), and Styrofoam (density 50 kg/m^3). You place the cubes in a large container filled with water. Rank the buoyant forces that the water exerts on the cubes from largest to smallest.

53. * You place four identical cubes made of oak (density 900 kg/m^3) in water, olive oil (density 880 kg/m^3), alcohol (density 790 kg/m^3), and mercury (density 13,600 kg/m^3). Rank the buoyant forces that the liquids exert on the cubes from largest to smallest.

54. ** You fill a 20-cm-tall container with glycerol so that the glycerol (density $1260\ kg/m^3$) reaches the 10-cm mark. You place an oak cube (density $900\ kg/m^3$) into the container. Each side of the cube is 10 cm. (a) What is the distance x from the upper face of the cube to the glycerol surface assuming the cube is in an upright position? (b) Make a prediction using qualitative reasoning (without mathematics) about what will happen to the cube in (a) if you now add olive oil (density $800\ kg/m^3$) to the container until it is completely full. Will the distance x decrease, increase, or stay the same? (c) Determine the new distance x using mathematics (physics laws) and compare your result with the prediction in (b).

55. * A 30-g ball with volume $37.5\ cm^3$ is attached to the bottom of a glass beaker with a light string. When the beaker is filled with water, the ball floats fully submerged under the water surface. Draw a force diagram for the ball and determine the force exerted on the ball by the string.

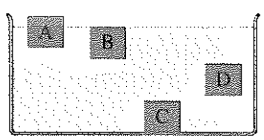
56. ** You have a ball (volume V, average density ρ_B) and a glass beaker. The ball is attached to the bottom of the beaker with a light spring (coefficient k). When you fill the beaker with a liquid of density ρ_L, the ball floats fully submerged under the surface with the spring extended by x. Draw a force diagram for the ball and derive the expression for x in terms V, ρ_B, ρ_L, and k. Evaluate the expression using unit analysis and limiting case analysis.

57. * This textbook says that the upward force that a fluid exerts on a submerged object is equal in magnitude to the product of the density of the fluid, the gravitational constant g, and the volume of the submerged part of the object. Where did this equation come from?

58. * **Design** This textbook says that the upward force that a fluid exerts on a submerged object is equal in magnitude to the product of the density of the fluid, the gravitational constant g, and the volume of the submerged part of the object. Design an experiment to test this expression, including a prediction about the outcome of the experiment.

59. * You have four objects at rest, each of the same volume. Object A is partially submerged, and objects B, C, and D are totally submerged in the same container of liquid, as shown in Figure P13.59. Draw a force diagram for each object. Rank the densities of the objects from least to greatest and indicate whether any objects have the same density.

FIGURE P13.59



60. * **Does air affect what a scale reads?** A 60-kg woman with a density of $980\ kg/m^3$ stands on a bathroom scale. Determine the reduction of the scale reading due to air.

61. * When analyzing a sample of ore, a geologist finds that it weighs 2.00 N in air and 1.13 N when immersed in water. What is the density of the ore? What assumptions did you make to answer the question? If the assumptions are not correct, how would the answer be different?

62. * A pin through a hole in the middle supports a meter stick. Two identical blocks hang from strings at an equal distance from the center so the stick is balanced. What happens to the stick if one block is submerged in water of density $1000\ kg/m^3$ and the other block in kerosene of density $850\ kg/m^3$?

63. * A meter stick is supported by a pin through a hole in the middle. (a) Two blocks made of the same material but different sizes hang from strings at different positions in such a way that the stick balances. What happens when the blocks hang entirely submerged in beakers of water? (b) Next you hang two blocks of different masses but the same volume at different positions so the stick balances. What happens when these blocks hang completely submerged in beakers of water? Support your answer for each part using force diagrams with arrows drawn with the correct relative lengths.

13.6 Skills for analyzing static fluid problems

64. **Goose on a lake** A 3.6-kg goose floats on a lake with 40% of its body below the $1000\text{-}kg/m^3$ water level. Determine the average density of the goose.

65. ** **Floating in seawater** A person of average density ρ_1 floats in seawater of density ρ_2. What fraction of the person's body is submerged? Explain.

66. * **Floating in seawater** A person of average density $980\ kg/m^3$ floats in seawater of density $1025\ kg/m^3$. What can you determine using this information? Determine it.

67. ** (a) Determine the force that a vertical string exerts on a 0.80-kg rock of density $3300\ kg/m^3$ when it is fully submerged in water of density $1000\ kg/m^3$. (b) If the force exerted by the string supporting the rock increases by 12% when the rock is submerged in a different fluid, what is that fluid's density? (c) If the density of another rock of the same volume is 12% greater, what happens to the buoyant force the water exerts on it?

68. * **Snorkeling** A 60-kg snorkeler (including snorkel, mask, and other gear) displaces $0.058\ m^3$ of water when 1.2 m under the surface. Determine the magnitude of the buoyant force exerted by the $1025\text{-}kg/m^3$ seawater on the person. Will the person sink or drift upward?

69. * A helium balloon of volume $0.12\ m^3$ has a total mass (the helium plus the balloon) of 0.12 kg. Determine the buoyant force exerted on the balloon by the air if the air has density $1.13\ kg/m^3$. Determine the initial acceleration of the balloon when released.

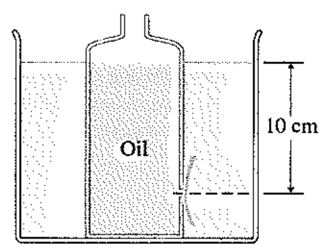
70. * BIO **Protein sinks in water** A protein molecule of mass 1.1×10^{-22} kg and density $1.3 \times 10^3\ kg/m^3$ is placed in a vertical tube of water of density $1000\ kg/m^3$. (a) Draw a motion diagram and a force diagram at the moment immediately after the molecule is released. (b) Determine the initial acceleration of the protein.

71. * How can you determine if a steel ball of known radius is hollow? List the equipment that you will need for the experiment, and describe the procedure and calculations. Can you determine how big the hollow part is if present in the ball?

72. ** **Crown composition** A crown is made of gold and silver. The scale reads its mass as 3.0 kg when in air and 2.75 kg when in water. Determine the masses of the gold and the silver in the crown. The density of gold is $19{,}300\ kg/m^3$ and that of silver is $10{,}500\ kg/m^3$.

73. * You place an open bottle filled with olive oil (density $880\ kg/m^3$) in a container filled with water so that the surfaces of both liquids are at the same level. The bottle has a hole in it 10 cm below the surface that is initially closed with adhesive tape (Figure P13.73). (a) Using Pascal's laws, predict what will happen when you remove the tape. Indicate any assumptions that you made. (b) Draw pressure-versus-depth graphs (similar to the graphs in Conceptual Exercise 13.3) for the states before and after you remove the tape. (c) If you predict any changes, determine their numerical values.

FIGURE P13.73

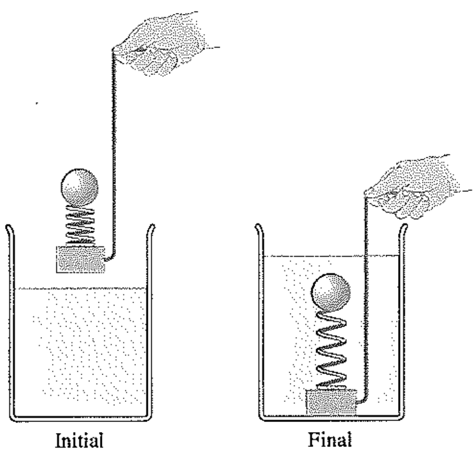


74. * You hold a ping-pong ball fully underwater (the initial state). When you release the ball, it jumps out of the water to a certain height above the surface. Let the final state be when the ball is just above the water surface, moving upward. Represent the process with energy bar charts (a) choosing the water, the ball, and Earth as the system, and (b) choosing the ball and Earth as the system. Indicate any assumptions that you made.

75. * You hang a steel ball on a string above a beaker that is filled to the top with water (the initial state). The beaker is sitting on a large empty tray. You slowly lower the ball until it reaches the bottom of the beaker. Some water spills over the rim of the beaker to the tray (the final state). (a) Represent the process with an energy bar chart choosing the water, the ball, and Earth as the system. (b) Repeat the analysis for a similar process with the same ball and the same amount of water, but a beaker tall enough so that no water spills over into the tray. Indicate any assumptions that you made.

76. * One end of a light spring is attached to a ping-pong ball, the other end to a heavy metal block that is fixed to a thin-wire holder (see Figure P13.76). You hold this setup so that the metal block is above the surface of water in a beaker (the initial state). You then slowly lower the setup until the metal block touches the bottom of the beaker (the final state). Represent the process with an energy bar chart, choosing the water, the ball, the spring, the block, and Earth as the system. Assume that the mass of the ball and the mass of the spring are much smaller than the mass of the block and so can be ignored. Indicate any other assumptions that you made.

FIGURE P13.76



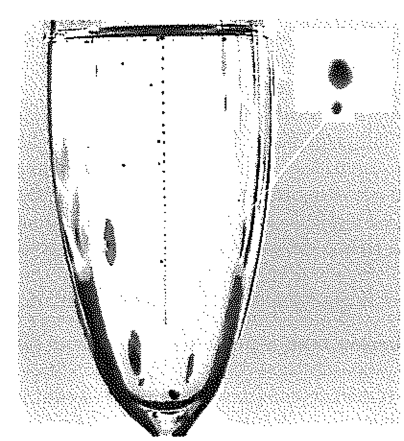
13.7 Ships, balloons, climbing, and diving

77. * **Wood raft** Logs of density 600 kg/m^3 are used to build a raft. What is the maximum mass of the load that can be supported by a raft built from 300 kg of logs?
78. * EST **Standing on a log** A floating log is L long and d in diameter. What is the mass of a person who can stand on the log without getting her feet wet?
79. * **Ferryboat** A ferryboat is 12 m long and 8 m wide. Two cars, each of mass 1600 kg, ride on the boat for transport across the lake. How much farther does the boat sink into the water?
80. EST **Iceberg** Icebergs are large pieces of freshwater ice. Estimate the percentage of the volume of an iceberg that is underwater. Indicate any assumptions that you made.
81. * **Life preserver** A life preserver is manufactured to support a 70-kg person with 20% of his volume out of the water. If the density of the life preserver is 100 kg/m^3 and it is completely submerged, what must its volume be? List your assumptions.
82. ** To increase the effect of the buoyant force on a submarine, the crew replaces seawater in the ballast tanks with air. To push the seawater out of the tanks, they use compressed air from air flasks. The submarine is located 10 m below the surface in seawater with density 1030 kg/m^3 and temperature 6 °C. How many kilograms of air should the crew let out from the flasks to increase the difference between the gravitational force and the buoyant force exerted on the submarine by 5×10^6 N? Assume the temperature of air in the flasks is the same as the seawater temperature.

General Problems

83. * Compare the density of water at 0 °C to the density of ice at 0 °C. Suggest possible explanations in terms of the molecular arrangements inside the liquid and solid forms of water that would account for the difference. If necessary, use extra resources to help answer the question.
84. * **Collapsing star** The radius of a collapsing star destined to become a pulsar decreases by 10% while at the same time 12% of its mass escapes. Determine the percent change in its density.
85. * **Deep dive** The *Trieste* research submarine traveled 10.9 km below the ocean surface while exploring the Mariana Trench in the South Pacific, the deepest place in the ocean. Determine the force needed to prevent a 0.10-m-diameter window on the side of the submarine from imploding. Assume that the pressure inside the submarine is 1 atm and the density of the water is 1025 kg/m^3.
86. * EST **Bursting a wine barrel** Pascal placed a long 0.20-cm-radius tube in a wine barrel of radius 0.24 m. He sealed the barrel where the tube entered it. When he added wine of density 1050 kg/m^3 to the tube so the column of wine was 8.0 m high, the cover of the barrel burst off the top of the barrel. Estimate the net force that caused the cover to come off.
87. * BIO **Lowest pressure in lungs** Experimentally determine the maximum distance you can suck water up a straw. Use this number to determine the pressure in your lungs above or below atmospheric pressure while you are sucking. Be sure to indicate any assumptions you made and show clearly how you reached your conclusion.
88. ** **Measuring ocean depth with a soda bottle** You can use an empty soda bottle with a cap to measure the depth to which you dive in the ocean. Dive to a certain depth, turn the closed bottle upside down (with the cap toward the sea bottom), and open the cap. A certain volume of seawater will enter the bottle, compressing the air that was trapped in it. Keep the bottle in an upside-down position and carefully close the bottle. Then swim up to the surface and measure the volume V_{water} of water in the bottle and the total volume V_0 of the bottle. (a) Derive the expression for the depth to which you dived in terms of V_{water}, V_0, and other relevant parameters. Indicate any assumptions that you made. Evaluate the expression using unit analysis and limiting case analysis.
89. * Justin is observing pearl-like strings of bubbles that move upward in his father's glass of champagne. He notices that as the bubbles rise, their volume increases (see Figure P13.89). He proposes the following explanation: "The pressure inside the bubble is equal to the pressure in the liquid surrounding the bubble. Because the pressure in the liquid decreases toward the surface of the liquid, the pressure in the ascending bubble decreases and therefore the volume of the bubble increases." Justin estimates that the size of a bubble near the surface is approximately twice that of a bubble 10 cm below the surface. Can Justin reject his explanation based on these data? Explain. If you answered yes, propose a different explanation that is consistent with the data and describe how you could test it. (Hint: Champagne contains dissolved carbon dioxide.)

FIGURE P13.89



90. ** You have an empty water bottle. Predict how much mass you need to add to it to make it float half-submerged. Then add the calculated mass and explain any discrepancy that you found. How did you make your prediction?
91. ** BIO **Flexible bladder helps fish sink or rise** A 1.0-kg fish of density 1025 kg/m^3 is in water of the same density. The fish's bladder contains 10 cm^3 of air. The bladder compresses to 4 cm^3. Now what is the density of the fish? Will it sink or rise? Explain.
92. * **Plane lands on Nimitz aircraft carrier** When a 27,000-kg fighter airplane lands on the deck of the aircraft carrier Nimitz, the carrier sinks 0.25 cm deeper into the water. Determine the cross-sectional area of the carrier.
93. Derive an equation for determining the unknown density of a liquid by measuring the magnitude of a force $T_{S\ on\ O}$ that a string needs to exert on a hanging object of unknown mass m and density ρ to support it when the object is submerged in the liquid.

Reading Passage Problems

BIO **Free diving** So-called "no-limits" free divers slide to deep water on a weighted sled that moves from a boat down a vinyl-coated steel cable to the bottom of a dive site. The diver reaches depths where a soda can would implode. After reaching the target depth, the diver releases the sled and an air bag opens and brings the diver quickly back to the surface. The divers have no external oxygen supply—just lungs full of air at the start of the dive. In August 2002, Tanya Streeter of the Cayman Islands held the women's no-limits free dive record at 160 m. In 2005 Patrick Musimu set the men's record with a 209.6-m free dive in the Red Sea just off the Egyptian coast (the record was later broken by Herbert Nitsch of Austria).

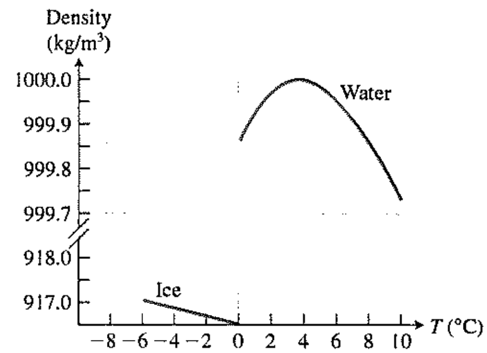
Musimu's 2005 dive took 3 minutes 28 seconds. He began the dive with his 9-L lungs full of air. By the time he passed the 200-m mark, Musimu's lungs had contracted to the size of a tennis ball. His body transferred blood from his limbs to essential organs such as the heart, lungs, and brain. This "blood shift" occurs when mammals submerge in water. Blood plasma fills the chest cavity, especially the lungs. Without this adaptation, the lungs would shrink and press against the chest walls, causing permanent damage. When he reached his target, Musimu released the weighted segment of the specialized sled that had taken him down and opened an airbag, which began his return to the surface at an average speed of 3–4 m/s.

94. Assuming Musimu weighs 670 N (150 lb) and is 1.6 m tall, 0.30 m wide, and 0.15 m thick, which answer below is closest to the magnitude of the force that the deep water exerted on one side of his body?
    (a) 0 (b) 670 N (130 lb)
    (c) 15,000 N (3000 lb) (d) 10^5 N (20,000 lb)
    (e) 10^6 N (200,000 lb)
95. Musimu's training allows him to hold up to 9 L = 9000 cm^3 of air when in a 1 atm environment. Which answer below is closest to the volume of that air if at pressure 22 atm?
    (a) 100 cm^3 (b) 200 cm^3 (c) 400 cm^3
    (d) 9000 cm^3 (e) 2×10^5 cm^3
96. As Musimu descends, the buoyant force that the water exerts on him
    (a) remains approximately constant.
    (b) increases a lot because the pressure is so much greater.
    (c) decreases significantly because his body is being compressed and made much smaller.
    (d) is zero for the entire dive.
    (e) There is not enough information to answer the question.
97. Why don't his lungs, heart, and chest completely collapse?
    (a) The return balloon helps counteract the external pressure.
    (b) There is no external force pushing directly on the organs.
    (c) The sled that helps him descend protects the front of his body.
    (d) Blood plasma moves from his extremities to his chest and the organs in it.
    (e) The air originally in the lungs is transferred to the vital organs.

Lakes freeze from top down We all know that ice cubes float in a glass of water. Why? Virtually every substance contracts when it solidifies—the solid is denser than the liquid. If this happened to water, ice cubes would sink to the bottom of a glass, and ice sheets would sink to the bottom of a lake. Fortunately, this doesn't happen. Liquid water expands by 9% when it freezes into solid ice at 0 °C, from a liquid density of a little less than 1000 kg/m^3 to a solid density of 917 kg/m^3.

But this is not the only special thing about water. While the density of most substances increases when they are cooled, water density shows a very peculiar temperature dependence (see Figure 13.17). As the temperature decreases, water density increases, but only until 4 °C, when water density reaches its maximum value, 1000.0 kg/m^3. As the temperature decreases further, water density decreases until at 0 °C water freezes and density abruptly decreases to 917 kg/m^3.

FIGURE 13.17

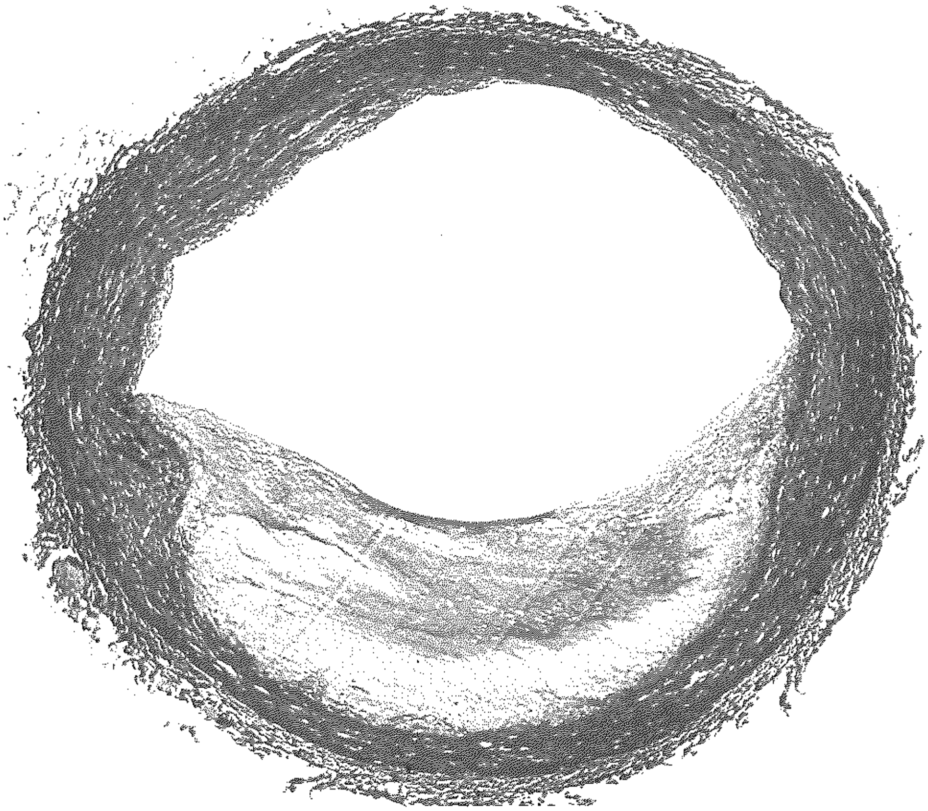


It is this peculiar water-density-temperature dependence that plays a vital role in the survival of animals and plants that live in water. In the winter when the water in a lake freezes, the solid ice stays at the top, forming an ice sheet. Water just below the ice sheet cools, but when it reaches 4 °C it becomes the most dense and sinks to the bottom of the lake. Since water colder than 4 °C is less dense, it stays above, keeping the bottom of the lake at a constant 4 °C. Note that if water were most dense at the freezing point, then in the winter the very cold water at the surface of lakes would sink. In this case the lake would freeze from the bottom up, and all life in it would be destroyed.

The expansion of water when it freezes has another important environmental benefit: the so-called freeze-thaw effect on sedimentary rocks. Water is absorbed into cracks in these rocks and then freezes in cold weather. The solid ice expands and cracks the rock, like a wood-cutter splitting logs. This continual process of liquid water absorption, freezing, and cracking releases mineral and nitrogen deposits into the soil and can eventually break the rock down into soil.

98. When is water densest?
    (a) When liquid at 0 °C
    (b) When solid ice at 0 °C
    (c) When liquid at 4 °C
    (d) Water density is always 1000.0 kg/m^3.
99. Why does water freeze from the top down?
    (a) The denser water at 0 °C sinks to the bottom of the lake.
    (b) The less dense ice at 0 °C rises above the liquid water at 0 °C.
    (c) The denser water at 4 °C sinks to the bottom of the lake.
    (d) Because of both a and b
    (e) Because of both b and c
100. Using Newton's second law, expressions for buoyant force and other forces, and the densities of liquid and solid water at 0 °C, find the fraction of an iceberg or an ice cube that is under liquid water.
    (a) 0.84 (b) 0.88 (c) 0.92
    (d) 0.96 (e) 1.00
101. A swimming pool at 0 °C has a very large chunk of ice floating in it—like an iceberg in the ocean. When the ice melts, what happens to the level of the water at the edge of the pool?
    (a) It rises. (b) It stays the same.
    (c) It drops. (d) It depends on the size of the chunk.
102. Which of the following is/are benefits of the temperature dependence of the density of water?
    (a) Fish and plants can survive winters without being frozen.
    (b) Over time, soil is formed from sedimentary rocks.
    (c) Water pipes when frozen in the winter do not burst.
    (d) Two of the above three
    (e) All of the first three

14



Fluids in Motion

Plaque (fatty deposits) accumulates on the walls of arteries as cholesterol-laden blood flows by. As plaque grows, blood flows past at higher speed. If the blood is moving fast enough, it can dislodge deposits, which may then become lodged downstream and stop blood flow. If this stoppage occurs in the heart, it can cause a heart attack. In this chapter, we will learn why blood flows faster through an artery clogged with plaque and why the fast-moving stream of blood tends to pull the plaque off the artery wall.

- **How does blood flow dislodge plaque from an artery?**
- **Why can a strong wind cause the roof to blow off a house?**
- **Why do people snore?**

IN THE PREVIOUS CHAPTER, we investigated the behavior of static fluids. What happens when a gas or liquid moves across a surface—for example, when air moves across the roof of a house or when blood moves through a blood vessel? In this chapter, we will investigate and explain phenomena involving moving fluids—*fluid dynamics*.

BE SURE YOU KNOW HOW TO:

- Draw work-energy bar charts (Section 7.2).
- Apply the concept of pressure to explain the behavior of liquids (Section 13.2).
- Draw force diagrams and apply Newton's second law (Section 3.5).

14.1 Fluids moving across surfaces—qualitative analysis

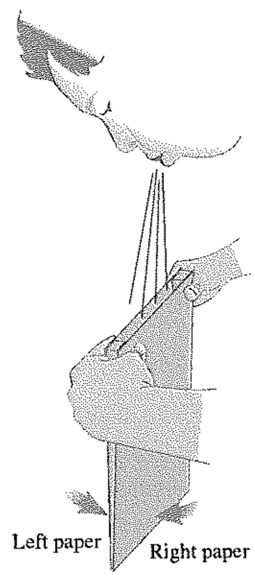
You learned in Chapters 12 and 13 that a key property of a static fluid is its pressure. What happens to that pressure when a fluid moves? To investigate this, we will analyze the simple experiment described in Observational Experiment Table 14.1.

OBSERVATIONAL EXPERIMENT TABLE 14.1 Pressure inside a moving fluid

VIDEO OET 14.1

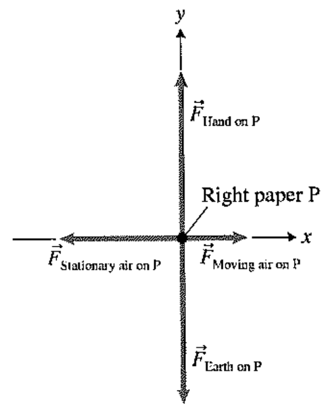
Observational experiment

Hold two pieces of paper separated by plastic blocks and blow down directly between them. You see the pieces of paper coming close together, as if they are pushed toward the moving air.



Analysis

We consider the piece of paper on the right to be our system and examine the moment when the paper starts moving to the left. We draw forces exerted on the paper in the horizontal direction. For the system to start moving, the sum of the horizontal forces must point to the left—from the region of stationary air to the region of moving air.



Pattern

Based on the analysis of the experiment and using the force diagram, we can infer that air moving along the surface of an object exerts a smaller force on the object. Given that force divided by the area over which it is exerted is pressure, we conclude that the air moving along a surface exerts less pressure on the object than stationary air.

Bernoulli's effect

Extrapolating from the pattern in Table 14.1, could it be that for any fluid, the speed with which fluid is moving along the object and the pressure exerted by this fluid on the object are related: the greater the speed, the smaller the pressure? Let's test this hypothesis experimentally.

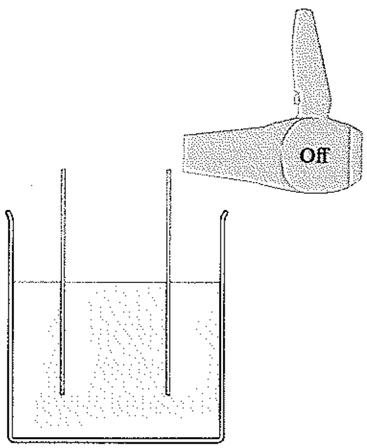
It is important to note here that in all of the experiments, the movement of the fluid was parallel to the surface of interest. When the fluid is moving perpendicular to the surface of interest, it exerts a force in the direction of motion (you can think of the momentum of fluid particles changing as the particles hit the surface). You can clearly observe this effect if you repeat the experiment in Testing Experiment Table 14.2 with the hairdryer blowing directly downward into the tube—the water level inside the tube lowers.

TESTING EXPERIMENT TABLE 14.2 How are the speed of a fluid and its pressure related?

VIDEO TET 14.2

Testing experiment

We cut the top and bottom off a plastic bottle to make a plastic tube and put it into a container of water. The level of the water in the tube is the same as in the container. Then using a hair dryer set to cold, we blow cold air parallel to the surface of the water right above the tube, first using a low-speed setting and then a high-speed setting.

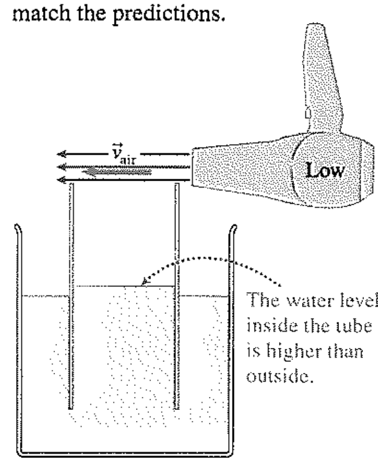


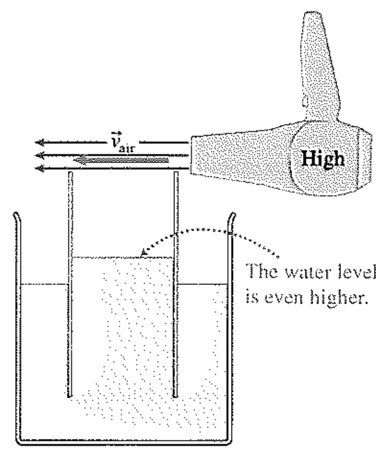
Prediction

If a faster-moving fluid exerts less pressure than a stationary fluid, and we blow air parallel to the opening of the tube, then the pressure at the top of the tube (but also inside the tube) should be lower than atmospheric pressure. When the speed of the moving air increases, the pressure should decrease. The pressure outside the tube, above the surface of the water in the container is atmospheric; therefore, the sum of the forces on an element of water inside the tube should point upward and the water in the tube should rise until the sum of the forces exerted on the raised water inside the tube is zero. The height of the water inside the tube should increase when the hair dryer is switched to high.

Outcome

The outcomes are shown in the figures. They match the predictions.





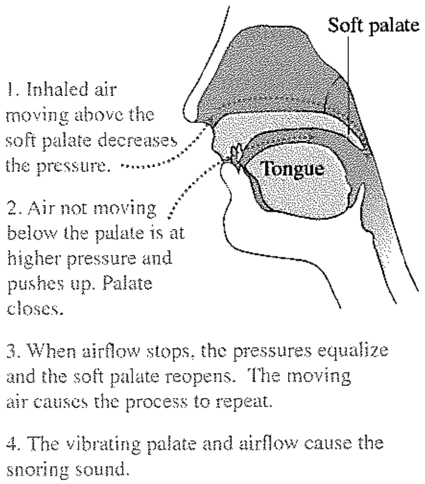
Conclusion

The hypothesis that the pressure exerted by a faster-moving fluid along an object is less than the pressure exerted by a slower-moving fluid was not disproved.

At this point, without evidence to the contrary, we can say that as a fluid's speed parallel to a surface increases, the pressure that the moving fluid exerts on the surface decreases. This statement is a qualitative version of a principle formulated in 1738 by Daniel Bernoulli and named in his honor.

Bernoulli's effect The pressure that a fluid exerts on a surface *decreases* as the speed with which the fluid moves parallel to the surface *increases*.

FIGURE 14.1 Snoring occurs when the soft palate opens and closes due to the starting and stopping of airflow across it.



Bernoulli's effect has important fluid-flow implications in biological systems—for example, in the flow of blood through blood vessels. The blood pressure against the wall of a vessel depends on how fast the blood is moving—pressure is lower when the blood is moving faster. Let's look at another biological application of Bernoulli's effect.

Snoring

A snoring sound occurs when air moving through the narrow opening above the soft palate at the back of the roof of the mouth has lower pressure than nonmoving air below the palate (Figure 14.1). The normal air pressure below the soft palate, where the air is not moving, pushes the palate closed. When airflow stops, the pressures equalize and the passage reopens. The rhythmic opening and closing of the soft palate against the throat leads to the snoring sound.

REVIEW QUESTION 14.1 What is the empirical evidence for Bernoulli's effect?

14.2 Flow rate and fluid speed

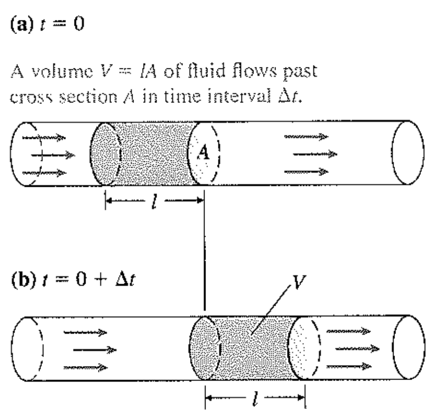
We have found qualitatively that the pressure of a fluid along the direction of flow depends on the speed of the moving fluid—the greater the speed, the lower the pressure. In this section, we will learn new physical quantities that we will need to describe the effect quantitatively. The quantitative analysis is much easier if we limit our investigations to fluids in confined regions—pipes or tubes.

Perhaps you have taken a shower in which little water flowed from the showerhead. In physics, we would say the water flow rate was low. The **flow rate Q** is an important consideration in designing showerheads. A smaller flow rate will save water, but a larger flow rate will help you rinse off faster. Flow rate is defined as the volume V of fluid that moves through a cross section of a pipe divided by the time interval Δt during which it moved (see Figure 14.2):

$$Q = \frac{V}{\Delta t} \tag{14.1}$$

The SI unit of flow rate is m^3/s, but you may also see it as ft^3/s, ft^3/min, gallons/min, L/min, or any unit of volume divided by any unit of time interval. Notice that flow rate in m^3/s is different from the speed of the fluid v in m/s.

FIGURE 14.2 Flow rate is the volume of fluid that passes a cross section of a vessel in a given time interval.



TIP The symbols V, t, and Q are also used in other aspects of physics. For example, a lowercase v denotes speed, the capital letter T is used for temperature, and in future chapters we will use Q for two other unrelated quantities. Because these symbols are often used to indicate different quantities, it is important when working with equations to try to visualize their meaning with concrete images (for example, the volume of water flowing out of a faucet during 1 s).

How does the flow rate relate to the speed of the moving fluid? To explore the relationship, consider Figure 14.2a. Over a certain time interval Δt the shaded volume of fluid passes a cross section of area A at some position along the pipe. Thus, after a time Δt, the back part of this fluid volume has in effect moved forward to the position shown in Figure 14.2b. The volume V of fluid in the shaded portion of the cylinder is the product of its length l and the cross-sectional area A of the pipe:

$$V = lA$$

Thus, the fluid flow rate is

$$Q = \frac{V}{\Delta t} = \frac{lA}{\Delta t} = \left(\frac{l}{\Delta t}\right)A$$

However, l is also the distance the fluid moves in a time interval Δt. Thus, $l/\Delta t$ is the average fluid speed v. Substituting $v = l/\Delta t$ into the above equation, we find that

$$Q = vA \qquad (14.2)$$

The flow rate is equivalent to the average fluid speed multiplied by the cross-sectional area of the pipe.

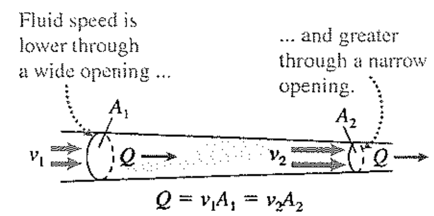
When an incompressible fluid (such as water) flows through a pipe with variable cross-sectional area (Figure 14.3), the amount of fluid entering at cross section A_1 must equal the amount of fluid leaving at cross section A_2. Thus the flow rate should remain constant. What must change is the speed of the fluid as it travels through the narrower part of the pipe. In the narrow section of the pipe (A_2), the speed will be greater in order to keep the flow rate constant. Thus, the flow rate past cross section A_1 equals that past cross section A_2:

$$Q_1 = v_1A_1 = v_2A_2 = Q_2 \qquad (14.3)$$

where v_1 is the average speed of the fluid passing cross section A_1, and v_2 is the average speed of the fluid passing cross section A_2. Equation (14.3) is called the **continuity equation** and is used to relate the cross-sectional area and average speed of fluid flow in different parts of a rigid pipe carrying an incompressible fluid.

TIP The definition of flow rate is an operational definition, not a cause-effect relationship, because it is the speed of the fluid that depends on the flow rate and the cross-sectional area. The flow rate is determined by the source of the fluid (for example, how much you open the faucet).

FIGURE 14.3 The flow speed $v_2 > v_1$ depends on the cross-sectional area of pipe carrying the fluid.



QUANTITATIVE EXERCISE 14.1 Speed of blood flow in the aorta

The heart pumps blood at an average flow rate of 80 cm^3/s into the aorta, which has a diameter of 1.5 cm. Determine the average speed of blood flow in the aorta.

Represent mathematically The flow rate can be determined by rearranging Eq. (14.2):

$$v = \frac{Q}{A}$$

where the cross-sectional area of the aorta is

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2$$

Solve and evaluate Combining the above two equations, we find that the average speed of blood flow in the aorta is

$$v = \frac{Q}{A} = \frac{Q}{\pi(d/2)^2} = \frac{(80\ \text{cm}^3/\text{s})}{\pi(1.5\ \text{cm}/2)^2} = 45\ \text{cm/s}$$

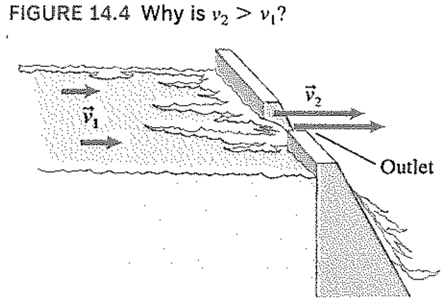
The unit is correct. The magnitude is reasonable—about half a meter each second.

Try it yourself Determine the average speed of blood flow if the diameter is reduced from 1.5 cm to 1.0 cm but the flow rate is the same.

Answer 100 cm/s.

Notice that blood speed more than doubles when the aorta diameter decreases by 33%. Vessel diameter has a very significant effect on the flow rate of fluid through a vessel, including those in biological systems. The narrower the blood vessel, the faster the blood flows, increasing the risk of dislodging plaque. Likewise, the narrower the airway from the nose to the mouth, the faster the air moves and the more likely you are to snore. These effects depend on the speed of the fluid, like blood or air in different parts of a vessel or pipe in which the diameter changes from one section to another.

FIGURE 14.4 Why is $v_2 > v_1$?



REVIEW QUESTION 14.2 Why does water in a river flow more slowly just before a dam than it does while passing through the outlet of the dam (Figure 14.4)?

FIGURE 14.5 Laminar flow.

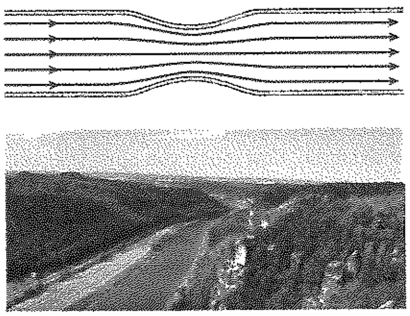


FIGURE 14.6 Turbulent flow.

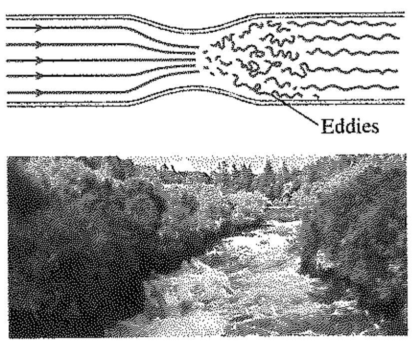


FIGURE 14.7 Streamlining a truck.

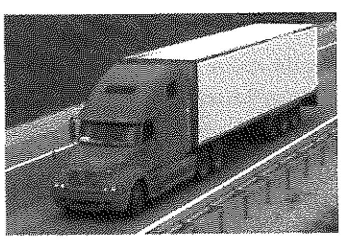
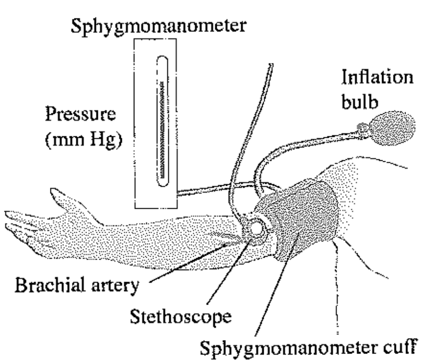


FIGURE 14.8 Measuring blood pressure



14.3 Types of fluid flow

Before we can proceed to describing Bernoulli's effect quantitatively, we need to learn more about fluid flow.

What kinds of flow can occur? There is the smooth flow that we see in a wide river and the whitewater flow that we see as water rushes and swirls through a narrow channel. Studies of fluid flow in wind tunnels indicate that there are two primary kinds of flow: laminar flow and turbulent flow. In **laminar flow**, every particle of fluid that passes a particular point follows the same path as the particles that preceded it. These paths are called **streamlines**. This is the smooth flow that we see in a wide river (see the fluid flowing smoothly in the wide tube in Figure 14.5). **Turbulent flow**, on the other hand, is characterized by agitated, disorderly motion. Instead of following a given path, the fluid forms whirlpool patterns called eddies, which come and go randomly, or sometimes become semi-stable (see Figure 14.6). Turbulent flow occurs when a fluid moves around objects and through pipes at high speed.

The force exerted by the fluid on objects is called the **drag force**. This is the force you "feel" when running against the wind. Due to the drag force, more kinetic energy is converted into thermal energy in turbulent flow than in laminar flow. Designing a car so that air moves over it with laminar flow reduces the drag force that the air exerts on the car and improves gasoline mileage. Placing a curved dome above the cab of a truck deflects air up and over the trailer, reducing turbulent flow and increasing gas mileage by more than 10% (see Figure 14.7).

Measuring blood pressure

To measure a person's blood pressure, a nurse uses a device called a *sphygmomanometer* (see Figure 14.8). The nurse places a cuff around the upper arm of a patient at about the level of the heart and places a stethoscope on the inside of the elbow above the brachial artery in the arm. The nurse then increases the gauge pressure in the cuff to about 180 mm Hg by pumping air into the cuff. The expanded cuff pushes on the brachial artery and stops blood flow in the arm. Then the nurse slowly releases the air from the cuff, decreasing the pressure of the air in it. When the pressure in the cuff is equal to the *systolic* pressure (120 mm Hg if the systolic blood pressure is normal), blood starts to squeeze through the artery past the cuff. The flow is intermittent and turbulent and causes a sound heard with the stethoscope. This turbulent sound continues until the cuff pressure decreases below the *diastolic* pressure (80 mm Hg for normal diastolic blood pressure). At that point the artery is continually open and blood flow is laminar and makes no sound. The systolic and diastolic pressure numbers together make up the blood pressure measurement.

REVIEW QUESTION 14.3 Is it easier for the heart to pump blood if the flow of the blood through the blood vessels is laminar or if it is turbulent? Explain.

14.4 Bernoulli's equation

Earlier in this chapter we investigated Bernoulli's effect qualitatively. In this section we will learn how to describe it quantitatively.

To achieve this goal, we use the case of a fluid flowing through a pipe, as shown in Figure 14.9a. We assume that (1) the fluid is incompressible, (2) there are no resistive forces exerted on the flowing fluid, and (3) the flow is laminar. We can apply the work-energy principle to describe the behavior of the fluid as it moves a short distance along the pipe. Consider the system to be composed of the shaded volume of fluid pictured in Figures 14.9a and b and Earth.

FIGURE 14.9 Applying the work-energy principle to fluid flow.

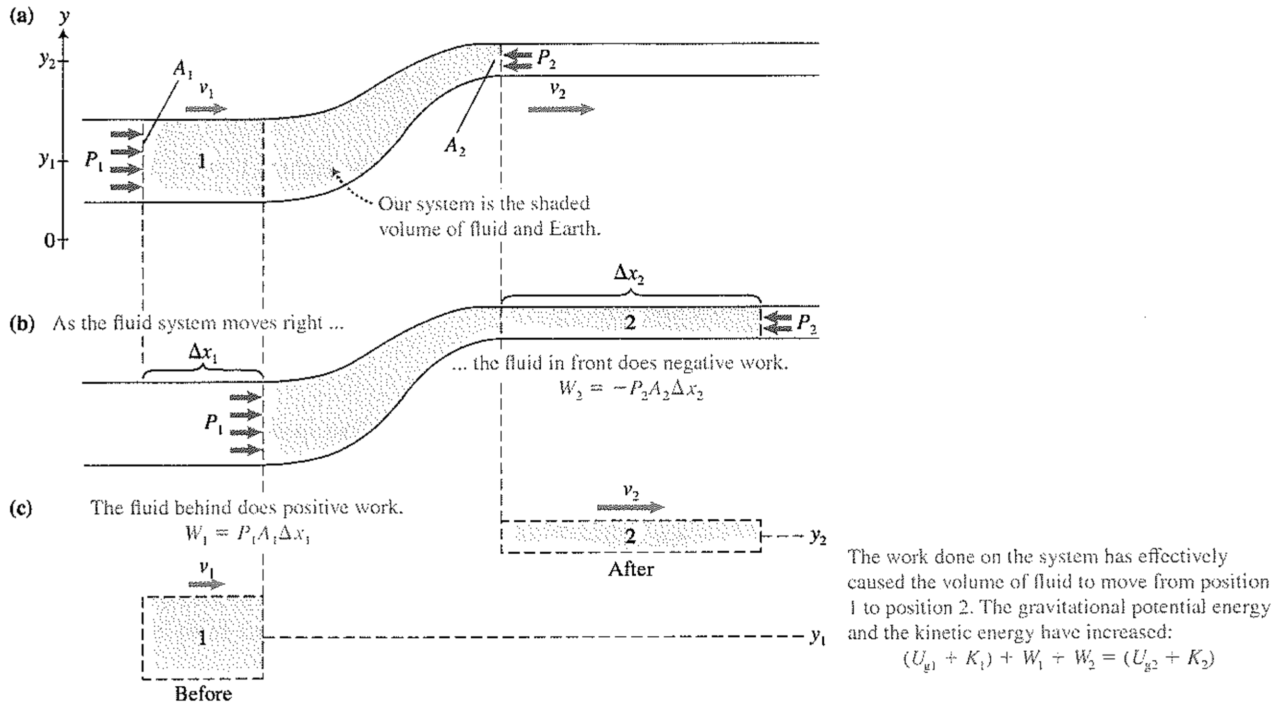


Figure 14.9a shows us the initial state of the system. As the volume of fluid flows to the right, the fluid behind it at position 1 exerts a force of magnitude $F_1 = P_1A_1$ to the right, where P_1 is the fluid pressure against the left side of the volume and A_1 is the cross-sectional area of the left side of the volume. Simultaneously, the fluid ahead of the system at position 2 exerts a force in the opposite direction of magnitude $F_2 = P_2A_2$, where P_2 is the fluid pressure against the right side of the volume and A_2 is the cross-sectional area of the right side of the volume.

In Figure 14.9b, the shaded volume of fluid has moved to the right. Because the pipe is narrower at 2, the right side of the fluid at position 2 moves a greater distance than the left side of the same volume of fluid in the wider part of the pipe at position 1. The net effect of the movement of the fluid a short distance to the right is summarized in Figure 14.9c. The volume of fluid initially at position 1 moving at speed v_1 has now been transferred, in effect, to position 2 where it moves at speed v_2. The volume of fluid stays constant, since we assume the fluid is incompressible. The fluid is now moving faster through the narrow tube at position 2 than it was earlier when moving through the wider tube at position 1, thus we have an increase in kinetic energy. The fluid at position 2 is at a higher elevation than when at position 1, thus the gravitational potential energy of the system increases. The energies changed as a result of the work done by the forces exerted by the fluid behind and ahead of the shaded volume. We can represent this quantitatively using the generalized work-energy equation [Eq. (7.3)].

$$(K_1 + U_{g1}) + W = (K_2 + U_{g2})$$

If we move the terms with the subscript 1 to the right side of the equation, we have

$$W = (K_2 - K_1) + (U_{g2} - U_{g1})$$

or

$$W = \Delta K + \Delta U_g \qquad (14.4)$$

Let us now write expressions for each of the terms in the above equation.

Work done Two forces are doing work on the system. The fluid behind the system exerts a force F_1 to the right on the left side of the system over a distance Δx_1. The fluid ahead of the system exerts a force F_2 to the left on the right side of the system over a distance Δx_2. (Figures 14.9a and b show fluid pressures; the corresponding forces have magnitudes $F_1 = P_1A_1$ and $F_2 = P_2A_2$.) The force F_1 does positive work since it points in the direction of the motion of the system. The force F_2 does negative work since it points in the direction opposite the motion of the system. The total work done on the system is

$$W = F_1\,\Delta x_1 \cos 0° + F_2\,\Delta x_2 \cos 180°$$
$$= P_1A_1\,\Delta x_1 - P_2A_2\,\Delta x_2$$

The volume of fluid V that has moved from the left to the right is $V = A_1\,\Delta x_1 = A_2\,\Delta x_2$ since the fluid is incompressible. The preceding expression for work becomes

$$W = P_1V - P_2V = (P_1 - P_2)V$$

Change in kinetic energy The mass m of the system (the moving volume of fluid) is related to its density ρ and volume V:

$$m = \rho V$$

As the system moves from the initial to the final state, the speed of the element of fluid that effectively moved changes from v_1 to v_2. Thus, the kinetic energy change of the mass m of fluid shown in Figure 14.9c is

$$\Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}\rho Vv_2^2 - \tfrac{1}{2}\rho Vv_1^2$$

Change in gravitational potential energy The gravitational potential energy of the system has also changed because part of the system has moved from elevation y_1 to elevation y_2. The change in gravitational potential energy is then

$$\Delta U_g = mg(y_2 - y_1) = \rho Vg(y_2 - y_1)$$

We can now substitute the above three expressions into Eq. (14.4) to get

$$(P_1 - P_2)V = (\tfrac{1}{2}\rho Vv_2^2 - \tfrac{1}{2}\rho Vv_1^2) + \rho Vg(y_2 - y_1)$$

If the common V is canceled from each term, we find that

$$P_1 - P_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

By dividing by V in that last step we have changed the units of each term in the equation from energy (measured in joules) to **energy density** (measured in joules per cubic meter). Energy density is the same as energy per unit volume of fluid and appears on the right side of the above equation. The left-hand side represents the amount of work done on the fluid per unit volume of fluid.

Bernoulli's equation relates the pressures, speeds, and elevations at two points on the same streamline in laminar flow in a fluid:

$$P_1 - P_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1) \qquad (14.5)$$

The equation can be rearranged into an alternate form:

$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \qquad (14.6)$$

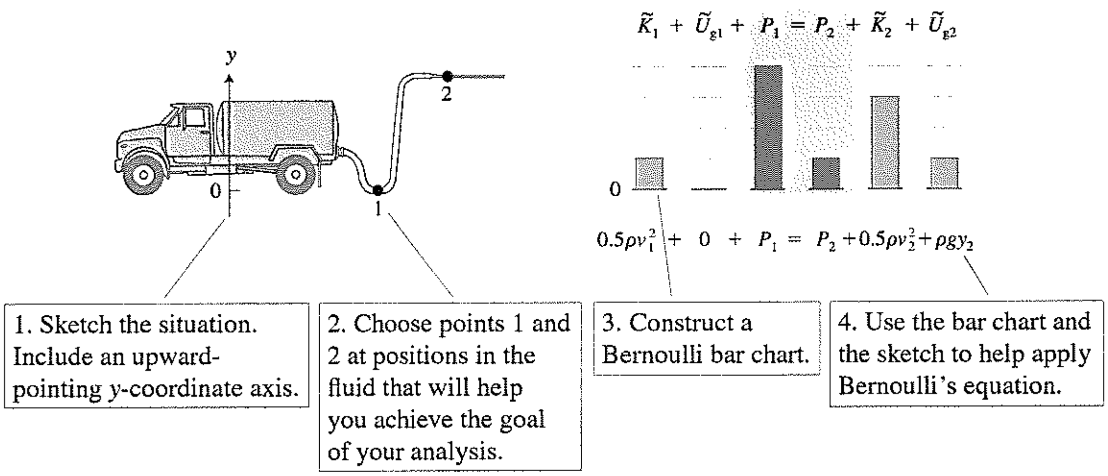
The sum of the kinetic and gravitational potential energy densities and the pressure at position 1 equals the sum of the same three quantities at position 2.

It is important to remember the assumptions that we used to derive Bernoulli's equation. It describes quantitatively the flow of a frictionless, nonturbulent, incompressible fluid.

Using Bernoulli bar charts to understand fluid flow

Bernoulli's equation looks fairly complex and might be difficult to use for visualizing fluid dynamics processes. However, since Bernoulli's equation is based on the work-energy principle, we can represent such processes using energy bar charts similar to the ones used in Chapters 7 and 13 (here the bars represent pressures and energy densities; we use a ~ over the energy symbols to emphasize that they are energy densities, not energies). Physics Tool Box 14.1 describes how to construct a fluid dynamics bar chart for the following process. A fire truck pumps water through a big hose up to a smaller hose on the ledge of a building. Water sprays out of the smaller hose onto a fire. Compare the pressure in the hose at the lowest point of the larger hose to the pressure at the exit of the smaller hose.

PHYSICS TOOL BOX 14.1 Constructing a bar chart for a moving fluid



To start, first draw a sketch of the process. Choose positions 1 and 2 at appropriate locations in order to help answer the question. One of the positions might be a place where you want to determine the pressure in the fluid and the other position a place where the pressure is known. For the water pump-hose process, it is useful to choose position 1 (the location of the unknown pressure) and position 2 at the exit of the water from the small hose (at known atmospheric pressure).

Represent this process by placing bars of appropriate relative lengths on the chart (the absolute lengths are not known). It is often easiest to start by analyzing the gravitational potential energy density. Use a vertical y-axis with a well-defined origin to keep track of the gravitational potential energy densities. For the fire hose process, choose position 1 as the origin of the vertical coordinate system. The gravitational potential energy density at position 1 is then zero. The exit of the water from the small hose is at higher elevation; thus, there is a positive gravitational potential energy density bar for position 2 (in the bar chart we arbitrarily assign it one positive unit of energy density).

Next, consider the kinetic energy. The water flows from a wider hose at position 1 to a narrower hose at position 2. Thus the kinetic energy density at 2 is greater than at 1—thus the longer bar at position 2. We arbitrarily assume that the kinetic energy density bar for position 1 is one unit and that for position 2 is three units.

Notice now that the total length of the bars on the right side of the chart is much higher than on the left side (the difference is three units). To account for the difference we need to consider the change in pressure. Since the fluid pressures at 1 and 2 are analogous to the work done on the system in an ordinary work-energy bar chart, P_1 and P_2 appear in the shaded box in the center of the bar chart, where work is represented. The difference in the pressure heights should account for the total difference in the energy densities. Thus, we draw the bar for P_1 three units higher than for P_2. The bar chart is now complete. We can use it to write a mathematical description for the process and solve for any unknown quantity.

REVIEW QUESTION 14.4 Compare and contrast work-energy bar charts and Bernoulli bar charts.

14.5 Skills for analyzing processes using Bernoulli's equation

In this section, we will adapt our problem-solving strategy to analyze processes involving moving fluids. In this case, we describe and illustrate a strategy for finding the speed of water as it leaves a bottle. The general strategy is on the left side of the table and the specific process is on the right.

PROBLEM-SOLVING STRATEGY 14.1 Applying Bernoulli's equation

EXAMPLE 14.2 Removing a tack from a water bottle

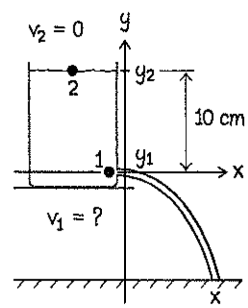
What is the speed with which water flows from a hole punched in the side of an open plastic bottle? The hole is 10 cm below the water surface.

Sketch and translate

- Sketch the situation. Include an upward-pointing y-coordinate axis. Choose an origin and positive direction for the coordinate axis.
- Choose points 1 and 2 at positions in the fluid where you know the pressure/speed/position or that involve the quantity you are trying to determine.
- Choose a system.

Example:

- Choose the origin of the vertical y-axis to be the location of the hole.
- Choose position 1 to be the place where the water leaves the hole and position 2 to be a place where the pressure, elevation, and water speed are known—at the water surface $y_2 = 0.10$ m. The pressure in Bernoulli's equation at both positions 1 and 2 is atmospheric pressure, since both positions are exposed to the atmosphere ($P_1 = P_2 = P_{atm}$).
- Choose Earth and the water as the system.

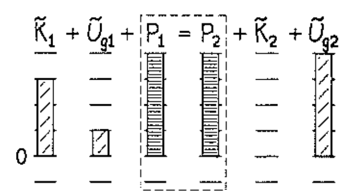


Simplify and diagram

- Identify any assumptions you are making. For example, can we assume that there are no resistive forces exerted on the flowing fluid?
- Construct a Bernoulli bar chart.

Example:

- Assume that no resistive forces are exerted on the flowing fluid.
- Assume that y_2 and y_1 stay constant during the process, since the elevation of the surface decreases slowly compared to the speed of the water as it leaves the tiny hole.
- Because the water at the surface is moving very slowly relative to the hole, assume that $v_2 = 0$.
- Draw a bar chart that represents the process.



Represent mathematically

- Use the sketch and bar chart to help apply Bernoulli's equation.
- You may need to combine Bernoulli's equation with other equations, such as the equation of continuity $Q = v_1 A_1 = v_2 A_2$ and the definition of pressure $P = F/A$.

Example:

- We see from the sketch and the bar chart that the speed of the fluid at position 2 is zero (zero kinetic energy density) and that the elevation is zero at position 1 (zero gravitational potential energy density). Also, the pressure is atmospheric at both 1 and 2. Thus

$$\frac{1}{2}\rho(0)^2 + \rho g y_2 + P_{atm} = P_{atm} + \frac{1}{2}\rho v_1^2 + \rho g(0)$$

$$\Rightarrow \rho g y_2 = \frac{1}{2}\rho v_1^2$$

Solve and evaluate

- Solve the equations for an unknown quantity.
- Evaluate the results to see if they are reasonable (the magnitude of the answer, its unit, how the answer changes in limiting cases, and so forth).

- Solve for v_1:

$$v_1 = \sqrt{2gy_2}$$

Substituting for g and y_2, we find that

$$v_1 = \sqrt{2(9.8\ \mathrm{m/s^2})(0.10\ \mathrm{m})} = 1.4\ \mathrm{m/s}$$

- The unit m/s is the correct unit for speed. The magnitude seems reasonable for water streaming from a bottle (if we obtained 120 m/s it would be unreasonably high).

Try it yourself In the above situation the water streams out of the bottle onto the floor a certain horizontal distance away from the bottle. The floor is 1.0 m below the hole. Predict this horizontal distance using your knowledge of projectile motion. (Hint: Use Eqs. (4.7) and (4.8).)

Answer The equations yield a result of 0.63 m. However, if we were to actually perform this experiment with a tack-sized hole, the water would land short of our prediction because of resistive forces exerted by the hole on the water. In order to make the water land closer to 0.63 m from the bottle, we must increase the diameter of the hole. We discuss the effect of resistive forces on fluid flow later in the chapter.

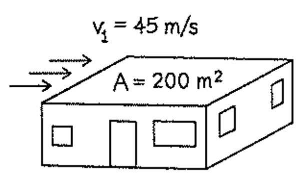
Blowing the roof off a house

You've no doubt seen images of roofs being blown from houses during tornadoes or hurricanes. How does that happen? On a windy day, the air inside the house is not moving, whereas the air outside is moving very rapidly. The air pressure inside the house is therefore greater than the air pressure outside, creating a net pressure that pushes outward against the roof and windows. If the net pressure becomes great enough, the roof and/or the windows will blow outward off of the house. In the following example, we do a quantitative estimate of the net force exerted by the inside and outside air on a roof.

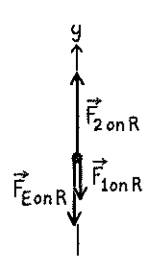
EXAMPLE 14.3 Effect of high-speed air moving across the roof of a house

During a storm, air moves at speed 45 m/s (100 mi/h) across the top of the 200-m^2 flat roof of a house. Estimate the net force exerted by the air pushing up on the inside of the roof and the outside air pushing down on the outside of the roof. Indicate any assumptions made in your estimate.

Sketch and translate The situation is shown at right. We need to determine the pressure just above and below the roof.



Simplify and diagram A force diagram for the roof is shown here. The air above the house exerts a downward force on the roof $F_{1\text{ on R}} = P_1A$, where P_1 is the air pressure above the house and A is the area of the roof. The air inside the house pushes up, exerting a force on the roof $F_{2\text{ on R}} = P_{\text{atm}}A$, where P_{atm} is the assumed atmospheric pressure of the stationary air inside the house. We assume that the air is incompressible and flows without friction or turbulence and that the roof is fairly thin so that the air has approximately the same gravitational potential energy density at points 1 and 2.



Represent mathematically With the y-axis oriented upward, the net force exerted by the air on the roof is

$$F_{\text{net Air}} = F_{2\text{ on R}} - F_{1\text{ on R}} = P_{\text{atm}}A - P_1A$$
$$= (P_{\text{atm}} - P_1)A$$

We use Bernoulli's equation to find this pressure difference.

$$P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 = \tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_1$$
$$\Rightarrow P_{\text{atm}} + 0 + \rho g y_2 = \tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_1$$
$$\Rightarrow P_{\text{atm}} - P_1 = \tfrac{1}{2}\rho v_1^2 + (\rho g y_1 - \rho g y_2) = \tfrac{1}{2}\rho v_1^2 + 0$$

We can now determine the net force exerted by the air on the roof.

$$F_{\text{net Air}} = (P_{\text{atm}} - P_1)A = \tfrac{1}{2}\rho v_1^2 A$$

(CONTINUED)

Solve and evaluate

$$F_{\text{net Air}} = \tfrac{1}{2}\rho v_1^2 A$$
$$= \tfrac{1}{2}(1.3\ \text{kg/m}^3)(45\ \text{m/s})^2(200\ \text{m}^2) = 2.6 \times 10^5\ \text{N}$$

The result is an upward net force that is enough to lift more than ten cars of combined mass 27,000 kg.

We were a little lax in applying Bernoulli's equation in this example. The equation relates the properties of a fluid at two points along the same streamline. A streamline does not flow between just below the roof and just above the roof. We could, with some more complex reasoning, use the equation correctly by considering two streamlines that start far from the house at the same pressure. One ends up in the house under its roof with the air barely moving. The other passes just above the roof with the air moving fast. We would get the same result in a somewhat more cumbersome manner.

Try it yourself A 2.0 m × 2.0 m canvas covers a trailer. The trailer moves at 29 m/s (65 mi/h). Determine the net force exerted on the canvas by the air above and below it.

Answer An upward force of 2200 N (about 500 lb). No wonder the canvas covering a truck trailer moving on a highway balloons outward.

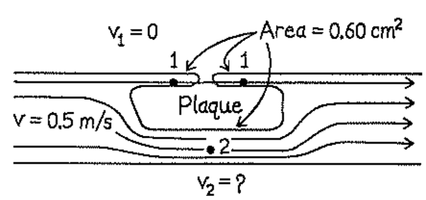
Dislodging plaque

The physical principles of a roof being lifted from a house also explain how plaque can become dislodged from the inner wall of an artery. Plaque may block a considerable portion of the area where blood normally flows. Suppose the radius of the vessel opening is one-third its normal value because of the plaque. Then the area available for blood flow, proportional to r^2, is about one-ninth the normal value. The speed of flow in the narrowed portion of the artery will be about nine times greater than in the unblocked part of the vessel. The kinetic energy density term in Bernoulli's equation is proportional to v^2 and therefore is 81 times greater in the constricted area than in the open part of the vessel.

Notice that in Bernoulli's equation, the sum of the gravitational potential energy density, the kinetic energy density, and the pressure at one location should equal the sum of the same three terms at some other location along a streamline in the blood. As blood speeds by the plaque, its kinetic energy density is 81 times greater, and consequently its pressure is much less than the pressure in the open vessel just before and just after the plaque. This pressure differential could cause the plaque to be pushed off the wall and tumble downstream, causing a blood clot (a process called thrombosis). Let's estimate the net force that the blood exerts on the plaque.

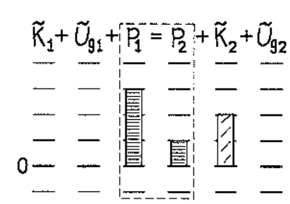
EXAMPLE 14.4 **A clogged artery**

Blood flows through the unobstructed part of a blood vessel at a speed of 0.5 m/s. The blood then flows past a plaque that constricts the cross-sectional area to one-ninth the normal value. The surface area of the plaque parallel to the direction of blood flow is about $0.60\ \text{cm}^2 = 6.0 \times 10^{-5}\ \text{m}^2$. Estimate the net force that the fluid exerts on the plaque.



Sketch and translate A simplified two-dimensional sketch of three-dimensional the situation is shown above right. The third dimension is perpendicular to the page of the book. The plaque is as large in this dimension as in the x-dimension, but the area where it is attached to the top wall is very small. Point 1 is above the plaque in stationary blood pooled in a channel where the plaque attaches to the artery wall. Point 2 is in the bloodstream below the plaque, where blood flows rapidly past it. The net force depends on the differences in pressure at points 1 and 2. Thus, we need to first find the pressure difference. Since points 1 and 2 are not on the same streamline, we cannot automatically use Bernoulli's equation. But since the streamlines that do go through points 1 and 2 were side by side before they reached the plaque, each streamline will have the same $P + (1/2)\rho v^2 + \rho g y$ value. This means we can equate the $P + (1/2)\rho v^2 + \rho g y$ values at points 1 and 2.

Simplify and diagram Assume for simplicity that the blood is nonviscous and flows with laminar flow without turbulence. Assume also that the vertical distance between points 1 and 2 is small ($y_1 - y_2 \approx 0$) and that the area of the stationary blood above the plaque is the same as the area where the blood moves below the plaque. A bar chart represents the process. The blood pressure at position 2 is less than at position 1 because the blood flows at high speed through the constricted artery, whereas it sits at rest in the channels at position $1(v_1 = 0)$.



Represent mathematically Compare the two points using Bernoulli's equation:

$$P_1 - P_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$
$$= \tfrac{1}{2}\rho(v_2^2 - 0) + 0 = \tfrac{1}{2}\rho v_2^2$$

Since the cross-sectional area of the vessel at the location of the plaque is one-ninth its normal area (the radius is one-third the normal value), the blood must be flowing at nine times its normal speed. The net force exerted by the blood on the plaque downward and perpendicular to the direction the blood flows will be

$$F_{\text{net blood on P }y} = F_{\text{blood 1 on P}} + (-F_{\text{blood 2 on P}})$$
$$= P_1A - P_2A = (P_1 - P_2)A$$
$$\Rightarrow F_{\text{net blood on P }y} = \tfrac{1}{2}\rho v_2^2 A$$

Solve and evaluate

$$F_{\text{net blood on P }y} = \tfrac{1}{2}(1050\,\text{kg/m}^3)\,(9 \times 0.5\,\text{m/s})^2(6.0 \times 10^{-5}\,\text{m}^2)$$
$$= 0.64\,\text{N} \approx 0.6\,\text{N}$$

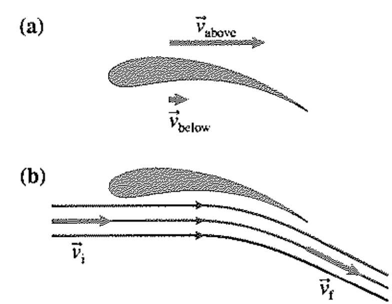
This is about the weight of one-half of an apple pulling on this tiny plaque. In addition, an "impact" force caused by blood hitting the plaque's upstream side contributes to the risk of breaking the plaque off the side wall of the vessel. The loose plaque can then tumble downstream and block blood flow in a smaller vessel in the heart (causing a heart attack) or in the brain (causing a stroke).

Try it yourself Air of density 1.3 kg/m^3 moves at speed 10 m/s across the top surface of a clarinet reed that has an area of 3 cm^2. The air below the reed is not moving and is at atmospheric pressure. Determine the net force exerted on the reed by the air above and below it.

Answer 0.02 N upward, toward the inside of the mouthpiece.

Using Bernoulli's equation to explain how airplanes fly

FIGURE 14.10 Airflow over an airplane wing.



You may have heard that airplanes can fly because of the special shape of their wings, which causes the air above the wing to move faster than the air below the wing ($\vec{v}_{above} > \vec{v}_{below}$, shown in Figure 14.10a). Let us apply Bernoulli's equation to an airplane wing to evaluate this claim. A Boeing 747-8's maximum takeoff mass is 442 metric tons, its takeoff speed is 330 km/h, and its wing surface area is 554 m^2. We assume that the speed of air above the wing is equal to the takeoff speed and the speed of air below the wing is zero (this is a rather unrealistic assumption, but it allows us to estimate the largest possible force due to Bernoulli's effect). In this case we find

$$\Delta p = \tfrac{1}{2}\rho_{\text{air}}v_{\text{takeoff}}^2 = 0.5 \times (1.3\,\text{kg/m}^3) \times (92\,\text{m/s})^2 = 5501\,\text{N/m}^2$$

and

$$F_{\text{air on airplane}} = \Delta p \cdot A = 3.1 \times 10^6\,\text{N}$$

The force exerted by Earth on the airplane is

$$F_{\text{E on airplane}} = m_{\text{airplane}}\,g = 4.3 \times 10^6\,\text{N}$$

This force is much larger than the largest possible lifting force exerted on the plane due to Bernoulli's effect.

In other words, Bernoulli's effect is at work, but it does not provide a complete explanation of how airplanes fly. The important missing force comes from the change of direction of the air's motion. Due to the shape and the tilt of the wing, the air passing the wing changes its direction of motion from horizontal ($\vec{v}_i$) to slightly downward ($\vec{v}_f$) (Figure 14.10b). For this to happen, the wing must exert a downward force on the air; therefore (remember Newton's third law), the air exerts an equal and opposite force upward on the wing. It is this force that provides the main lift and (together with the force due to Bernoulli's effect) makes the airplane fly. Note that the origin of this force is the change of the direction of motion of air and has nothing to do with Bernoulli's effect.

REVIEW QUESTION 14.5 In Example 14.2 we said that the pressure was the same at two levels when we drew the bar chart. Doesn't the pressure in a fluid increase with depth?

14.6 Viscous fluid flow

In our previous discussions and examples in this chapter, we assumed that there were no resistive forces exerted on the moving fluid or that fluids flow without friction. That is, we assumed no interaction either between the fluid and the walls of the pipes it flows in, or between the layers of the fluid. However, in Example 14.2 we found that this assumption was not reasonable. In fact, for many processes, such as the transport of blood in the small vessels in our bodies, fluid friction is very important. When we cannot neglect this friction inside the fluid, we call the fluid **viscous**.

Consider the following situation. You have an object that can slide on a frictionless horizontal surface, say, a puck on smooth ice. You push the puck abruptly and then let go. What happens to the puck? Once in motion, the puck will continue to slide at constant speed with respect to the ice even if nothing else pushes it. However, if there is friction between the contacting surfaces (there is a little sand in the ice), then the puck starts slowing down; for it to continue moving at constant speed, someone or something has to push it forward to balance the opposing friction force.

By analogy, if a fluid flows through a horizontal tube without friction (nonviscous fluid), we would expect it to continue to flow at a constant rate with no additional forward pressure. But if friction is present (the fluid is viscous), there must be greater pressure at the back of the fluid than at the front of the fluid to maintain a constant flow rate. If this is the case, the force exerted on any volume of the fluid due to the forward pressure is greater than the force exerted on the same volume of the fluid due to the pressure in the opposite direction.

Factors that affect fluid flow rate

What factors affect the flow rate in the vessel with friction? What is the functional dependence of those factors? Let's think about the physical properties of the fluid and the vessel that can affect the flow rate. The following quantities might be important.

Pressure difference The flow rate should depend on how hard the fluid is pushed forward, that is, on the difference between the fluid pressure pushing forward from behind and the fluid pressure pushing back from in front of the fluid, or $(P_1 - P_2)$.

Radius of the tube The radius r of the tube carrying the fluid should affect the flow rate. From everyday experience we know that it is more difficult to push (a greater pressure difference) fluid through a tube of tiny radius than through a tube with a large radius.

Length of the tube The length l of the tube might also affect the ease of fluid flow. A long tube offers more resistance to flow than a shorter tube.

Fluid type Water flows much more easily than molasses does. Thus some property of a fluid that characterizes its "thickness" or "stickiness" should affect the flow.

Let's design an experiment to investigate exactly how the first three of these four factors (P, r, and l) affect the fluid flow rate Q. As shown in Figure 14.11, a pump that produces an adjustable pressure P_1 causes fluid to flow through tubes of different radii r and lengths l. We collect the fluid exiting the tube and measure the flow rate Q, which is the volume V of fluid leaving the tube in a certain time interval Δt divided by that time interval. The results of the experiments are reported in Table 14.3.

How is the flow rate affected by each of the three factors?

Pressure difference Looking at the first three rows of the table, we notice that the flow rate is proportional to the pressure difference ($Q \propto P_1 - P_2$).

FIGURE 14.11 How do $P_1 - P_2$, r, and l affect the flow rate Q?

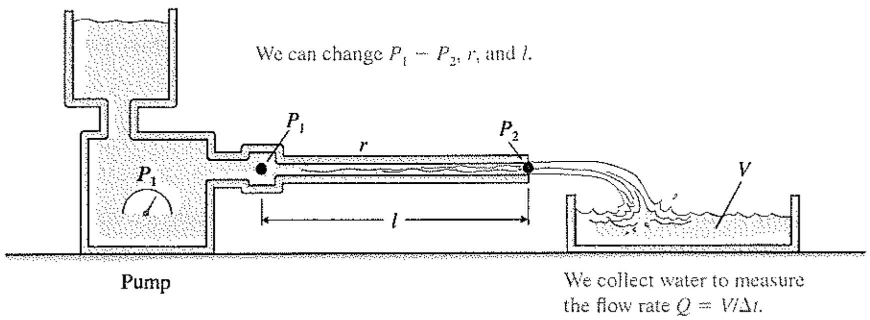


TABLE 14.3 Different quantities affect the flow rate Q of fluid through a tube (data are reported in relative units)

$P_1 - P_2$ (Pressure difference)	r (Radius)	l (Length)	Q (Flow rate)
1	1	1	1
2	1	1	2
3	1	1	3
1	2	1	16
1	3	1	81
1	1	2	0.5
1	1	3	0.33

Radius of the tube Looking at rows 1, 4, and 5, we notice that the flow rate increases rapidly as the radius increases. Doubling the radius causes the flow rate to increase by a factor of 16 (2^4). Tripling the radius causes the flow rate to increase by a factor of 81 (3^4). It seems that the flow rate is proportional to the fourth power of the radius of the tube ($Q \propto r^4$).

Length of the tube Looking at rows 1, 6, and 7, we notice that the flow rate decreases as the length of the tube increases. It seems that the flow rate is proportional to the inverse of the length ($Q \propto 1/l$).

These three relationships can be combined in a single equation:

$$Q \propto \frac{r^4(P_1 - P_2)}{l}$$

The equation has been confirmed by numerous experiments.

Viscosity and Poiseuille's law

In this experiment we did not investigate the fourth factor: the type of fluid. Under the same conditions, water flows faster than oil, which flows faster than molasses. If we use the same pressure difference to push different fluids through the same tube, we find that the fluids have different flow rates. The quantity by which we measure this effect on flow rate is called the **viscosity** η of the fluid. The flow rate is inversely proportional to viscosity:

$$Q \propto \frac{1}{\eta}$$

In 1840, using an experiment similar to that described above, French physician and physiologist Jean Louis Marie Poiseuille established a relationship between these physical quantities. However, instead of writing the flow rate in terms of the other four quantities, he wrote an expression for the pressure difference needed to cause a particular flow rate.

Poiseuille's law The forward-backward pressure difference $P_1 - P_2$ needed to cause a fluid of viscosity η to flow at a rate Q through a vessel of radius r and length l is

$$P_1 - P_2 = \left(\frac{8}{\pi}\right)\frac{\eta l}{r^4}Q \qquad (14.7)$$

The pressure difference term $P_1 - P_2$ on the left side of Poiseuille's law determines the net force pushing the fluid. The flow rate Q on the far right side is a consequence of this net push on the fluid. The term before Q on the right side (the $\left(\frac{8}{\pi}\right)\frac{\eta l}{r^4}$ term) can be thought of as the resistance of the fluid to flow: the same pressure difference will produce a smaller flow rate if the resistance is greater. The resistance is greater if the fluid has greater viscosity η, is greater for a longer tube (greater l), and is far more resistive if the vessel through which the fluid flows has a smaller radius ($1/r^4$ is much greater

TIP Notice that the pressure difference needed to cause a particular flow rate is proportional to the inverse of the fourth power of the radius of the vessel. If the radius of a vessel carrying fluid is reduced by a factor of 0.5, the pressure difference needed to cause the same flow rate must increase by $(1/0.5)^4 = 16$. We need 16 times the pressure difference to cause the same flow rate.

TABLE 14.4 Viscosities of some liquids and gases

Substance	Viscosity η ($N \cdot s/m^2$)
Air (30 °C)	1.9×10^{-5}
Water vapor (30 °C)	1.25×10^{-5}
Water (0 °C)	1.8×10^{-3}
Water (20 °C)	1.0×10^{-3}
Water (40 °C)	0.66×10^{-3}
Water (80 °C)	0.36×10^{-3}
Blood, whole (37 °C)	4×10^{-3}
Oil, SAE No. 10	0.20

for small r). This idea has many applications relative to the circulatory system—see the example a little later in this section.

From Poiseuille's law we can determine the unit for viscosity. To do this, we express the viscosity using other quantities in Eq. (14.7):

$$P_1 - P_2 = \left(\frac{8}{\pi}\right)\frac{\eta l}{r^4}Q \Longrightarrow \eta = \frac{(P_1 - P_2)r^4\pi}{8Ql}$$

We use the latter equation to find the units for the viscosity. Remember that the units of pressure are

$$\text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{m}^2} = \frac{\text{kg}}{\text{s}^2\cdot\text{m}}$$

and the units for flow rate are m^3/s. Using these units, we get

$$\eta = \frac{(\text{kg})(\text{m}^4)(\text{s})}{(\text{s}^2\cdot\text{m})(\text{m}^3)(\text{m})} = \frac{\text{kg}}{\text{s}\cdot\text{m}}$$

We can also rewrite the last combination of units as $\text{N}\cdot\text{s}/\text{m}^2$. Remember that $\text{N}/\text{m}^2 = \text{Pa}$; thus the unit for viscosity can be written $\text{Pa}\cdot\text{s}$. A list of viscosities of several fluids appears in Table 14.4 using $\text{N}\cdot\text{s}/\text{m}^2$ for the units of η.

QUANTITATIVE EXERCISE 14.5 Blood flow through a narrow artery

Because of plaque buildup, the radius of an artery in a person's heart decreases by 40%. Determine the ratio of the present flow rate to the original flow rate if the pressure across the artery, its length, and the viscosity of blood are unchanged.

Represent mathematically In this exercise, we are interested in the change in the flow rate and not in the change in pressure. Consequently, we rearrange Poiseuille's law for the flow rate in terms of the other quantities:

$$Q = \left(\frac{\pi}{8}\right)\left(\frac{\Delta P}{\eta l}\right)r^4$$

If the radius decreases by 40%, the new radius is 100% − 40% = 60% of the original. Thus the radius r of the vessel at the present time is related to the radius r_0 years earlier by the equation $r = 0.60r_0$.

Solve and evaluate The ratio of the flow rates is

$$\frac{Q}{Q_0} = \frac{\left(\frac{\pi}{8}\right)\left(\frac{\Delta P}{\eta l}\right)r^4}{\left(\frac{\pi}{8}\right)\left(\frac{\Delta P}{\eta l}\right)r_0^4} = \frac{r^4}{r_0^4} = \left(\frac{r}{r_0}\right)^4 = (0.60)^4 = 0.13$$

The flow rate is only 13% of the original flow rate! To compensate for such a dramatically reduced flow rate, the person's blood pressure will increase.

Try it yourself Determine the reduction in flow rate, assuming a constant pressure difference, if the radius of the vessel is reduced 90% (to 0.10 times its original value). This is not an unusual reduction for people with high blood pressure.

Answer $Q/Q_0 = 0.0001$, or 0.01% of its original value!

Limitations of Poiseuille's law: Reynolds number

Poiseuille's law describes the flow of a fluid accurately only when the flow is laminar. Experiments indicate that to determine when the flow is laminar or turbulent, we need to calculate what is called the pipe **Reynolds number** R_e:

$$R_e = \frac{2\bar{v}r\rho}{\eta} \tag{14.8}$$

where $\bar{v}$ is the average speed of the fluid, ρ is its density, η is the viscosity, and r is the radius of the pipe that carries the fluid. Experiments show that if the Reynolds number is less than 2000, the fluid flow is laminar; if it is more than 3000, the flow is turbulent; and between 2000 and 3000 the flow is unstable and can be either laminar or turbulent.

The Reynolds number, used as a criterion to decide whether the flow inside a pipe is laminar or turbulent, came from experiments conducted in the 1880s by Osborne Reynolds. Using a glass pipe with flowing water inside, he adjusted a valve to control the speed of the water flow. He then added colored water to the stream and observed that when the speed of the water was low, the colored layer of water could be clearly seen inside the pipe. When the speed of water flow increased beyond a certain limit, the colored part would break apart into vortices and mix with the rest of the water. Reynolds expressed the criterion for the type of flow with a unitless number (hence the name the Reynolds number), which he derived by taking the ratio of the net force pushing the fluid and the resistive force opposing it.

The transition from laminar to turbulent flow is also observed when an object and fluid are moving relative to each other. Reynolds derived a similar criterion (expressed as a unitless number) for this phenomenon. In general, the flow in pipes remains laminar up to much larger R_e numbers than does the flow of a fluid around an object.

REVIEW QUESTION 14.6 Describe some of the physics-related effects on the cardiovascular system of medication that lowers the viscosity of blood.

14.7 Drag force

So far in this chapter all of our analyses have focused on a moving fluid. In Section 14.6 we were concerned with the resistive forces as fluids move through a tube. Now we focus on solid objects moving through a fluid—for example, a swimmer moving through water, a skydiver falling through the air, or a car traveling through air. As you know from experience, the fluid in these and in other cases exerts a resistive **drag force** on the object moving through the fluid. So far we have been neglecting this force in our mechanics problems. Now we will not only learn how to calculate this force but also learn whether our assumptions were reasonable: for example, is the resistive drag force indeed insignificant when people and objects fall from small and large heights?

Laminar drag force Imagine that an object moves relatively slowly through a fluid (for example, a marble sinking in oil). In this case the fluid flows around the solid object in streamline laminar flow, with no turbulence. However, the fluid does exert a drag force on the object. For a spherical object O of radius r moving at speed v through a liquid of viscosity η, the magnitude of this nonturbulent drag force $F_{\text{D F on O}}$ exerted by fluid on the object is given by the equation

$$F_{\text{D F on O}} = 6\pi\eta r v \tag{14.9}$$

This equation is called **Stokes's law**. Notice that the drag force is proportional to the speed of the object relative to the fluid and to the radius of the sphere.

Turbulent drag force A rock sinks in oil much faster than a small marble. In this case the motion of oil past the sinking rock is turbulent, and Eq. (14.9) does not apply. A different Reynolds number, called the object Reynolds number, can be used to determine whether the flow of fluid past an object is laminar or turbulent:

$$R_e = \frac{vl\rho}{\eta} \tag{14.10}$$

where v is the object's speed with respect to the fluid, l is the characteristic dimension of the object, in most cases the diameter, and ρ and η are the density and viscosity of the fluid. When the Reynolds number is calculated using this equation, the threshold value for laminar flow is 1. If the Reynolds number is much more than 1, the flow is turbulent and we cannot use Stokes's law. In this case, a new equation for drag force applies:

$$F_{\text{D F on O}} \approx \tfrac{1}{2} C_{\text{D}} \rho A v^2 \tag{14.11}$$

where ρ is the density of fluid, A is the cross-sectional area of the object as seen along its line of motion, and C_{D} is a dimensionless number called the *drag coefficient*. The drag coefficient depends on the shape of the object (the lower the number, the smaller the drag force and the more laminar the flow past the object). For example, the drag coefficient for a sphere is 0.5 and for a dolphin it is 0.005.

Drag force exerted on a moving vehicle

Does Stokes's law apply to moving cars? At 60 mi/h (about 27 m/s), for a car about 2 m wide in air with density 1.3 kg/m^3 and viscosity $2 \times 10^{-5}\ \text{N} \cdot \text{s/m}^2$, the estimated Reynolds number is

$$R_{\text{c}} = \frac{vl\rho}{\eta} = \frac{(27\ \text{m/s})(2\ \text{m})(1.3\ \text{kg/m}^3)}{(2 \times 10^{-5}\ \text{N} \cdot \text{s/m}^2)} \approx 4 \times 10^6$$

This is much more than 1. We need to use Eq. (14.11) for the drag force.

TIP Note that if the speed of a car doubles, the drag force exerted on it quadruples. Thus, because of air drag, when you increase your driving speed, you reduce your gas mileage.

We can estimate the magnitude of the drag force that air exerts on a compact car traveling at 27 m/s (60 mi/h) assuming that the drag coefficient C_{D} is approximately 0.3 for a well-designed car, the cross-sectional area of a car is 2 m^2, and the air density is 1.3 kg/m^3.

Because the flow of air past the car is turbulent, we use Eq. (14.11) to estimate the drag force that the air exerts on the car:

$$F_{\text{D Air on Car}} = \tfrac{1}{2} C_{\text{D}} \rho A v^2 = \tfrac{1}{2}(0.3)(1.3\ \text{kg/m}^3)(2\ \text{m}^2)(27\ \text{m/s})^2 = 280\ \text{N}$$

or a force of about 60 lb. Designing cars to minimize the drag force improves fuel efficiency.

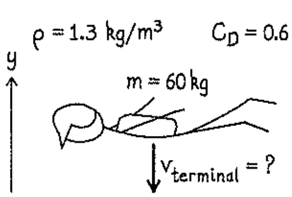
Terminal speed

As a skydiver falls through the air, her speed increases, as does the drag force that the air exerts on her. Eventually, the diver's speed becomes so great that the magnitude of the upward resistive drag force that the air exerts on the diver equals the magnitude of the downward gravitational force that Earth exerts on the diver. The sum of the forces exerted on the diver is zero, so the diver moves downward at a constant speed, known as **terminal speed**. Let's estimate the terminal speed for a skydiver.

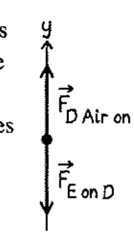
EXAMPLE 14.6 Terminal speed of skydiver

Estimate the terminal speed of a 60-kg skydiver falling through air of density 1.3 kg/m^3, assuming a drag coefficient $C_{\text{D}} = 0.6$.

Sketch and translate The situation is sketched here. When the diver is moving at terminal speed, the forces that the air exerts on the diver and that Earth exerts on the diver balance—the net force is zero. We choose the diver as the system of interest with vertical y-axis pointing upward.



Simplify and diagram A force diagram for the diver is shown to the right. Assume that the buoyant force that the air exerts on the diver is negligible in comparison to the other forces exerted on her and that the drag force involves turbulent airflow past the diver.



Represent mathematically Use the force diagram to help apply Newton's second law for the diver:

$$ma_y = \Sigma F_y$$

$$\Rightarrow 0 = F_{\text{D Air on D}} + (-F_{\text{E on D}})$$

$$= \tfrac{1}{2} C_{\text{D}} \rho A v_{\text{terminal}}^2 - mg$$

Solve and evaluate Solving for the diver's terminal speed gives

$$v_{\text{terminal}} = \sqrt{\frac{2mg}{C_D \rho A}}$$

All of the quantities in the above expression are known except the cross-sectional area of the diver along her line of motion. If we assume that she is 1.5 m tall and 0.3 m wide, her cross-sectional area is about 0.5 m^2. We find that her terminal speed is

$$v_{\text{terminal}} = \sqrt{\frac{2(60\text{ kg})(9.8\text{ N/kg})}{(0.6)(1.3\text{ kg/m}^3)(0.5\text{ m}^2)}} = 55\text{ m/s}$$

The unit is correct. The magnitude seems reasonable—about 120 mi/h.

Note that we assumed a turbulent drag force (Eq. (14.11). Was this assumption appropriate? To assess this, we will estimate the maximum speed of the diver for the drag force to be laminar. According to Eq. (14.10),

$$R_e = \frac{vl\rho}{\eta} \quad \Rightarrow \quad v = \frac{R_e \eta}{l\rho}$$

Estimating the characteristic dimension for the person to be about 1 m (somewhere between her 1.5-m height and 0.3-m width) and the critical value of the Reynolds number to be 1 for a laminar drag force, we obtain the following estimate for the greatest speed of an object for which the drag force exerted by the fluid is laminar:

$$v = \frac{R_e \eta}{l\rho} \approx \frac{1 \times (1.9 \times 10^{-5}\text{ N}\cdot\text{s/m}^2)}{(1\text{ m}) \times (1.3\text{ kg/m}^3)} \approx 1.5 \times 10^{-5}\text{ m/s}$$

This is a tiny speed—much too slow for a skydiver. Therefore, we could safely use the equation for turbulent drag force to estimate her speed.

Try it yourself Suppose the diver pulled her legs to her chest so she was more in the shape of a ball. How qualitatively would that affect her terminal speed? Explain.

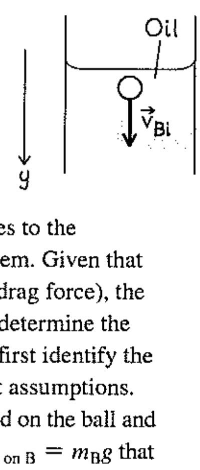
Answer Her cross-sectional area would be smaller, and according to the above equation, her terminal speed would be greater.

Below is an example of a problem whose solution depends on our assumptions. Being able to consider the effects of assumptions is a very useful skill.

EXAMPLE 14.7 Multiple possibility problem

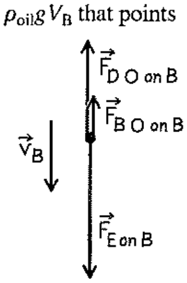
A small metal ball is launched vertically downward, with initial velocity $\vec{v}_{Bi}$, from just below the surface of oil that fills a very deep container. Draw a qualitative velocity-versus-time and an acceleration-versus-time graph for the motion of the ball.

Sketch and translate We draw a sketch of the situation; let's choose the y-axis to point down. In order to draw velocity- and acceleration-versus-time graphs, we need to know the sum of the forces exerted on the ball.



Simplify and diagram The term "simplify" relates to the assumptions that we need to make to solve the problem. Given that one of the forces exerted on the ball is variable (the drag force), the assumptions we make about the initial velocity will determine the outcome. Therefore, in this case it is more useful to first identify the forces exerted on the ball and then consider different assumptions.

First, we need to identify the forces that are exerted on the ball and draw a force diagram (at right). Earth exerts a force $F_{E\text{ on }B} = m_B g$ that points down. The oil exerts a buoyant force $F_{B\text{ O on }B} = \rho_{oil} g V_B$ that points up. Since the ball is made of metal we know that $F_{E\text{ on }B} > F_{B\text{ O on }B}$. The oil also exerts a drag force $F_{D\text{ O on }B}$ that is proportional to v_B and points opposite to $\vec{v}_B$ (up in our case). Given that we are interested in qualitative graphs, it is enough to remember that the drag force exerted by the oil on the ball is proportional to the speed of the ball with respect to the oil. This is true for a laminar as well as for a turbulent drag force.

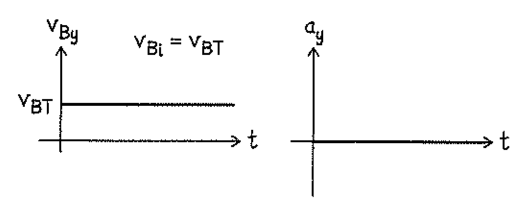


Since $a_y = \dfrac{\Sigma F_{O\text{ on }B\,y}}{m_B}$, and $F_{E\text{ on }B\,y}$ and $F_{B\text{ O on }B\,y}$ are constant, the sign and magnitude of $\vec{a}_y$ will depend on the magnitude of $F_{D\text{ O on }B\,y}$ or on the speed of the ball. Let us consider the following assumptions.

1. The ball is initially moving such that the magnitude of the drag force is exactly equal to the difference between the gravitational force and the buoyant force:

   $$F_{D\text{ O on }B\,y} = F_{E\text{ on }B\,y} - F_{B\text{ O on }B\,y} \Rightarrow \Sigma F_{O\text{ on }B\,y} = 0;\ a_y = 0$$

   In this special case, the ball will continue to move with the same speed with which it was launched. This is the terminal speed, which we will denote v_{BT}. The graphs for this motion are shown below.



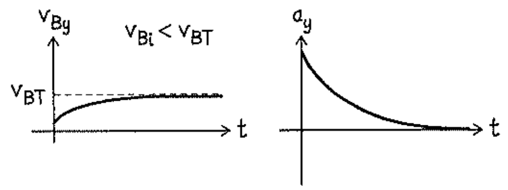
2. The ball is initially moving such that the magnitude of the drag force is *smaller* than the difference between the gravitational force and the buoyant force:

   $$F_{D\text{ O on }B\,y} < F_{E\text{ on }B\,y} - F_{B\text{ O on }B\,y} \Rightarrow \Sigma F_{O\text{ on }B\,y} > 0;\ a_y > 0$$

   Since $F_{D\text{ O on }B}$ is proportional to v_B, this case will occur when $v_{Bi} < v_{BT}$. In this case, the ball will initially accelerate downward,

(CONTINUED)

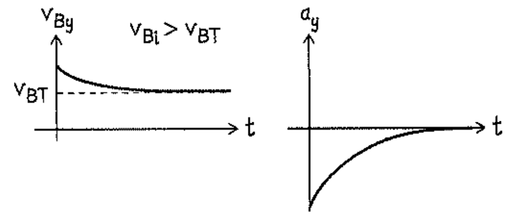
but as the speed of the ball increases, $F_{\text{D O on B}}$ will increase until the sum of the forces is zero. As explained in case 1, at this point the ball reaches terminal velocity and continues moving with constant speed (zero acceleration). The graphs for this motion are shown below.



3. The ball is initially moving such that the magnitude of the drag force is *larger* than the difference between the gravitational force and the buoyant force:

$$F_{\text{D O on B}\,y} > F_{\text{E on B}\,y} - F_{\text{B O on B}\,y} \Rightarrow \Sigma F_{\text{O on B}\,y} < 0\,;\, a_y < 0$$

Since $F_{\text{D O on B}}$ is proportional to v_{B}, this case will occur when $v_{\text{Bi}} > v_{\text{BT}}$. In this case, the acceleration of the ball will initially point up (the ball will slow down). As the speed of the ball decreases, $F_{\text{D O on B}}$ will decrease until the sum of the forces is zero. Therefore, the ball will slow down until it reaches the terminal velocity and then continues moving with constant speed (zero acceleration). The graphs for this motion are shown below.



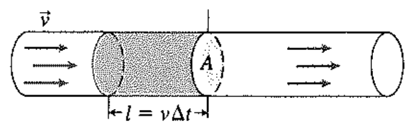
Try it yourself Describe what will happen if we shoot the ball upward from the bottom of the oil-filled container. How does the scenario depend on the initial speed of the ball?

Answer No matter what the initial speed of the ball, the ball will move upward and slow down because all the forces exerted on it point in the direction opposite to its motion. Then it will start moving downward in the same way as described in case 2 in this example.

REVIEW QUESTION 14.7 When a skydiver falls at constant terminal speed, shouldn't the magnitude of the resistive drag force that the air exerts on the skydiver be a little less than the magnitude of the downward gravitational force that Earth exerts on the diver? If they are equal, shouldn't the diver stop falling? Explain.

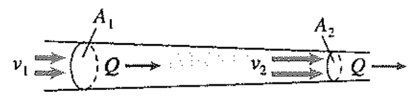
Summary

Flow rate The flow rate Q of a fluid is the volume V of fluid that passes a cross section in a tube divided by the time interval Δt needed for that volume to pass. The flow rate also equals the product of the average speed v of the fluid and the cross-sectional area A of the vessel. (Section 14.2)



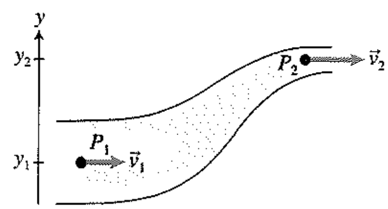
$$Q = vA \qquad \text{Eq. (14.2)}$$

Continuity equation If fluid does not accumulate, the flow rate into a region (position 1) must equal the flow rate out of the region (position 2). At position 1 the fluid has speed v_1 and the tube has cross-sectional area A_1. At position 2 the fluid has speed v_2 and the tube has cross-sectional area A_2. (Section 14.2)



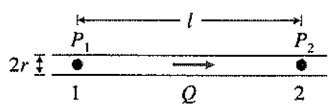
$$Q = v_1 A_1 = v_2 A_2 \qquad \text{Eq. (14.3)}$$

Bernoulli's equation For a fluid flowing without resistive forces or turbulence, the sum of the kinetic energy density $(1/2)\rho v^2$, the gravitational potential energy density $\rho g y$, and pressure P of the fluid is a constant. (Section 14.4)



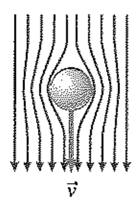
$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \qquad \text{Eq. (14.6)}$$

Poiseuille's law For viscous fluid flow, the pressure drop $(P_1 - P_2)$ across a fluid of viscosity η flowing in a tube depends on the length l of the tube, its radius r, and the fluid flow rate Q. (Section 14.6)



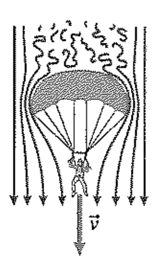
$$P_1 - P_2 = \left(\frac{8}{\pi}\right)\frac{\eta l}{r^4}Q \qquad \text{Eq. (14.7)}$$

Laminar drag force When a spherical object (like a marble sinking in oil) moves slowly through a fluid, the fluid exerts a resistive drag force on the object that is proportional to the object's speed v. **Stokes's law** describes the force. (Section 14.7)



$$F_D = 6\pi\eta r v \qquad \text{Eq. (14.9)}$$

Turbulent drag force For an object moving at faster speed through a fluid (like a parachutist with an open parachute), turbulence occurs and the resistive drag force is proportional to the square of the speed. (Section 14.7)



$$F_D = \tfrac{1}{2}C_D \rho A v^2 \qquad \text{Eq. (14.11)}$$

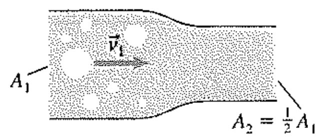
Questions

Multiple Choice Questions

1. A roof is blown off a house during a tornado. Why does this happen?
   (a) The air pressure in the house is lower than that outside.
   (b) The air pressure in the house is higher than that outside.
   (c) The wind is so strong that it blows the roof off.
2. A river flows downstream and widens, and the flow speed slows. As a result, the pressure of the water against a dock downstream compared to upstream will be
   (a) higher. (b) lower. (c) the same.
3. Why does the closed top of a convertible bulge when the car is riding along a highway?
   (a) The volume of air inside the car increases.
   (b) The air pressure is greater outside the car than inside.
   (c) The air pressure inside the car is greater than the pressure outside.
   (d) The air blows into the front part of the roof, lifting the back part.

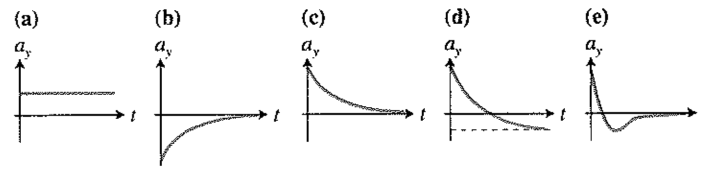
4. How does Bernoulli's principle help explain air going up the chimney of a house?
   (a) Air blowing across the top of the chimney reduces the pressure above the chimney.
   (b) The air above the chimney attracts the ashes.
   (c) The hot ashes seek the cooler outside air.
   (d) The gravitational potential energy is lower above the chimney.
5. As a river approaches a dam, the width of the river increases and the speed of the flowing water decreases. What can explain this effect?
   (a) Bernoulli's equation
   (b) The continuity equation
   (c) Poiseuille's law
6. What is an incompressible fluid?
   (a) A law of physics
   (b) A physical quantity
   (c) A model of an object
7. What is viscous flow?
   (a) A physical phenomenon
   (b) A law of physics
   (c) A physical quantity
8. The heart does about 1 J of work pumping blood during one heartbeat. What is the immediate first and main type of energy that increases due to the heart's work?
   (a) Kinetic energy
   (b) Thermal energy
   (c) Elastic potential energy
9. Several air bubbles are present in water flowing through a pipe of variable cross-sectional area (Figure Q14.9). What happens to the volume of an air bubble when it arrives at the narrow part of the pipe, where the cross-sectional area is half that of the wider part? Assume the temperature of the gas in the bubble stays constant.

FIGURE Q14.9



   (a) The volume of the bubble remains unchanged because water is incompressible.
   (b) The volume of the bubble remains unchanged because the temperature of the gas in the bubble is constant.
   (c) The volume of the bubble decreases because the pressure in water increases.
   (d) The volume of the bubble increases because the pressure in water decreases.
10. A small metal ball is released from just below the surface of oil that fills a very deep container. The y-axis points down. Which of the acceleration-versus-time graphs in Figure Q14.10 represents the motion of the ball after it is released?

FIGURE Q14.10



11. A small metal ball is launched downward from just below the surface of oil that fills a very deep container. The initial speed of the ball is larger than the terminal speed of the ball in the oil. The y-axis points down. Which of the graphs in Figure Q14.10 represents the motion of the ball after it is launched?

Conceptual Questions

12. You have two identical large jugs with small holes on the side near the bottom. One jug is filled with water and the other with liquid mercury. The liquid in each jug, sitting on a table, squirts out the side hole into a container on the floor. Which container, the one catching the water or the one catching the mercury, must be closer to the table in order to catch the fluid? Or should they be placed at the same distance? Which jug will empty first, or do they empty at the same time? Explain. Indicate any assumptions that you made.
13. Why does much of the pressure drop in the circulatory system occur across the arterioles (small vessels carrying blood to the capillaries) and capillaries as opposed to across the much larger diameter arteries?
14. If you partly close the end of a hose with your thumb, the water squirts out farther. Give at least one explanation for why this phenomenon occurs.
15. Compare and contrast work-energy bar charts, which you learned about in Chapter 7, with Bernoulli bar charts.
16. Consider Bernoulli's equation, Poiseuille's law, and Stokes's law. Which of these are applicable to viscous fluids? Explain.
17. You need a liquid that will exhibit turbulent flow in a tube even at lower speeds. Which properties of liquids will you evaluate when choosing a liquid? Explain.

Problems

Below, BIO indicates a problem with a biological or medical focus. Problems labeled EST ask you to estimate the answer to a quantitative problem rather than derive a specific answer. Asterisks indicate the level of difficulty of the problem. Problems with no * are considered to be the least difficult. A single * marks moderately difficult problems. Two ** indicate more difficult problems. Unless stated otherwise, assume in these problems that atmospheric pressure is $1.01 \times 10^5\ N/m^2$ and that the densities of water and air are $1000\ kg/m^3$ and $1.3\ kg/m^3$, respectively.

14.1 and 14.2 Fluids moving across surfaces—qualitative analysis and Flow rate and fluid speed

1. **Watering plants** You water flowers outside your house. (a) Determine the flow rate of water moving at an average speed of 32 cm/s through a garden hose of radius 1.2 cm. (b) Determine the speed of the water in a second hose of radius 1.0 cm that is connected to the first hose.
2. **Irrigation canal** You live near an irrigation canal that is filled to the top with water. (a) It has a rectangular cross section of 5.0-m width and 1.2-m depth. If water flows at a speed of 0.80 m/s, what is its flow rate? (b) If the width of the stream is reduced to 3.0 m and the depth to 1.0 m as the water passes a flow-control gate, what is the speed of the water past the gate?
3. **Fire hose** During a fire, a firefighter holds a hose through which $0.070\ m^3$ of water flows each second. The water leaves the nozzle at an average speed of 25 m/s. What information about the hose can you determine using these data?
4. The main waterline for a neighborhood delivers water at a maximum flow rate of $0.010\ m^3/s$. If the speed of this water is 0.30 m/s, what is the pipe's radius?
5. * BIO **Blood flow in capillaries** The average flow rate of blood in the aorta is $80\ cm^3/s$. Beyond the aorta, this blood eventually travels through about 6×10^9 capillaries, each of radius 8.0×10^{-4} cm. What is the average speed of the blood in the capillaries?
6. * **Irrigating a field** It takes a farmer 2.0 h to irrigate a field using a 4.0-cm-diameter pipe that comes from an irrigation canal. How long would the job take if he used a 6.0-cm pipe? What assumption did you make? If this assumption is not correct, how will your answer change?

14.4 Bernoulli's equation

7. Represent the process sketched in Figure P14.7 using a qualitative Bernoulli bar chart and an equation (include only terms that are not zero).

FIGURE P14.7

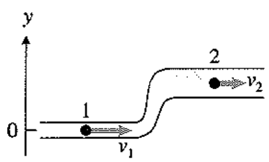
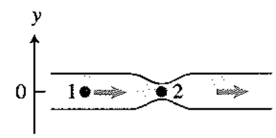


FIGURE P14.8



8. * Represent the process sketched in Figure P14.8 using a qualitative Bernoulli bar chart and an equation (include only terms that are not zero).
9. **Fluid flow problem** Write a symbolic equation (include only terms that are not zero) and draw a sketch of a situation that could be represented by the qualitative Bernoulli bar chart shown in Figure P14.9 (there are many possibilities).

FIGURE P14.9

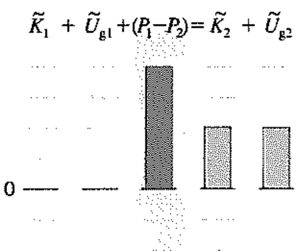
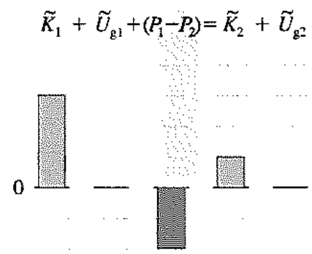


FIGURE P14.10



10. Repeat Problem 14.9 using the bar chart in Figure P14.10.
11. * Repeat Problem 14.9 using the bar chart in Figure P14.11.

FIGURE P14.11

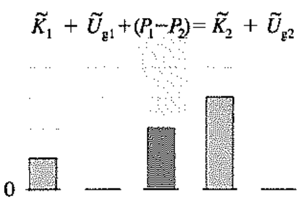
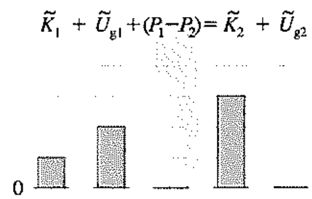


FIGURE P14.12



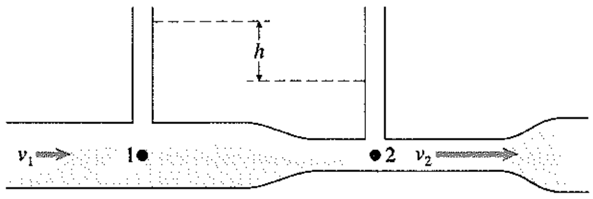
12. Repeat Problem 14.9 using the bar chart in Figure P14.12.
13. An application of Bernoulli's equation is shown below. Construct a qualitative Bernoulli bar chart that is consistent with the equation and draw a sketch of a situation that could be represented by the equation (there are many possibilities).
$\rho g y_2 = 0.5\rho v_1^2$
14. Repeat Problem 14.13 using the equation $0.5\rho v_1^2 + (P_1 - P_2) = 0.5\rho v_2^2$ and $P_1 < P_2$.
15. * Repeat Problem 14.13 using the equation below.
$0.5\rho v_1^2 + (P_1 - P_2) =$
$0.5\rho v_2^2 + \rho g y_2$ and $P_1 < P_2$.
16. * **Wine flow from barrel** While visiting a winery, you observe wine shooting out of a hole in the bottom of a barrel. The top of the barrel is open. The hole is 0.80 m below the top surface of the wine. Represent this process in multiple ways (a sketch, a bar chart, and an equation) and apply Bernoulli's equation to a point at the top surface of the wine and another point at the hole in the barrel.
17. **Water flow in city water system** Water is pumped at high speed from a reservoir into a large-diameter pipe. This pipe connects to a smaller diameter pipe. There is no change in elevation. Represent the water flow from the large pipe to the smaller pipe in multiple ways—a sketch, a bar chart, and an equation.

14.5 Skills for analyzing processes using Bernoulli's equation

18. * The pressure of water flowing through a 0.060-m-radius pipe at a speed of 1.8 m/s is $2.2 \times 10^5\ \text{N/m}^2$. What is (a) the flow rate of the water and (b) the pressure in the water after it goes up a 5.0-m-high hill and flows in a 0.050-m-radius pipe?
19. * **Siphoning water** You want to siphon rainwater and melted snow from the cover of an above-ground swimming pool. The cover is 1.4 m above the ground. You have a plastic hose of 1.0-cm radius with one end in the water on the pool cover and the other end on the ground. (a) At what speed does water exit the hose? (b) If you want to empty the pool cover in half the time, what new hose radius should you use? (c) How much faster does the water flow through this wider pipe?
20. * **Cleaning skylights** You are going to wash the skylights in your kitchen. The skylights are 8.0 m above the ground. You connect two garden hoses together—a 0.80-cm-radius hose to a 1.0-cm-radius hose. The smaller hose is held on the roof of the house and the wider hose is attached to the faucet on the ground. The pressure at the opening of the smaller hose is 1 atm, and you want the water to have the speed of 6.0 m/s. What should be the pressure at ground level in the large hose? What should be the speed?
21. * BIO **Blood flow in artery** Blood flows at an average speed of 0.40 m/s in a horizontal artery of radius 1.0 cm. The average pressure is $1.4 \times 10^4\ \text{N/m}^2$ above atmospheric pressure (the gauge pressure). (a) What is the average speed of the blood past a constriction where the radius of the opening is 0.30 cm? (b) What is the gauge pressure of the blood as it moves past the constriction? What assumption did you make to answer these questions?
22. * **Straw aspirator** A straw extends out of a glass of water by a height h. How fast must air blow across the top of the straw to draw water to the top of the straw?
23. * **Gate for irrigation system** You observe water at rest behind an irrigation dam. The water is 1.2 m above the bottom of a gate that, when lifted, allows water to flow under the gate. Determine the height h from the bottom of the dam that the gate should be lifted to allow a water flow rate of $1.0 \times 10^{-2}\ \text{m}^3/\text{s}$. The gate is 0.50 m wide.
24. * BIO **Flutter in blood vessel** A person has a 5200-N/m^2 gauge pressure of blood flowing at 0.50 m/s inside a 1.0-cm-radius main artery. The gauge pressure outside the artery is $3200\ \text{N/m}^2$. When using his stethoscope, a physician hears a fluttering sound farther along the artery. The sound is a sign that the artery is vibrating open and closed, which indicates that there must be a constriction in the artery that has reduced its radius and subsequently reduced the internal blood pressure to less than the external 3200-N/m^2 pressure. What is the maximum artery radius at this constriction?
25. * BIO **Effect of smoking on arteriole radius** The average radius of a smoker's arterioles, the small vessels carrying blood to the capillaries, is 5% smaller than those of a nonsmoker. (a) Determine the percent change in flow rate if the pressure across the arterioles remains constant. (b) Determine the percent change in pressure if the flow rate remains constant.
26. * **Roof of house in wind** The mass of the roof of a house is 2.1×10^4 kg and the area of the roof is 160 m^2. At what speed must air move across the roof of the house so that the roof is lifted off the walls? Indicate any assumptions you made.
27. * You have a U-shaped tube open at both ends. You pour water into the tube so that it is partially filled. You have a fan that blows air at a speed of 10 m/s. (a) How can you use the fan to make water rise on one side of the tube? Explain your strategy in detail. (b) To what maximum height can you get the water to rise? Note: You cannot touch the water yourself.

28. * Engineers use a venturi meter to measure the speed of a fluid traveling through a pipe (see Figure P14.28). Positions 1 and 2 are in pipes with surface areas A_1 and A_2, with A_1 greater than A_2, and are at the same vertical height. How can you determine the relative speeds at positions 1 and 2 and the pressure difference between positions 1 and 2?

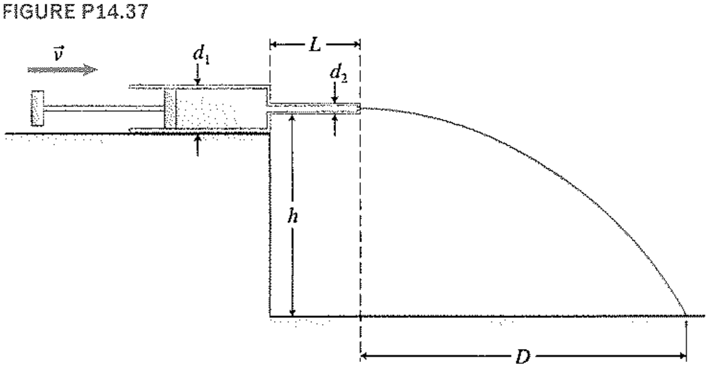
FIGURE P14.28



14.6 Viscous fluid flow

29. * A 5.0-cm-radius horizontal water pipe is 500 m long. Water at 20 °C flows at a rate of 1.0×10^{-2} m^3/s. (a) Determine the pressure drop due to viscous friction from the beginning to the end of the pipe. (b) What radius pipe must you use if you want to keep the pressure difference constant and double the flow rate?
30. **Fire hose** A volunteer firefighter uses a 5.0-cm-diameter fire hose that is 60 m long. The water moves through the hose at 12 m/s. The temperature outside is 20 °C. What is the pressure drop due to viscous friction across the hose?
31. **Another fire hose** The pump for a fire hose can develop a maximum pressure of 6.0×10^5 N/m^2. A horizontal hose that is 50 m long is to carry water of viscosity 1.0×10^{-3} N·s/m^2 at a flow rate of 1.0 m^3/s. What is the minimum radius for the hose?
32. * **Solar collector water system** Water flows in a solar collector through a copper tube of radius R and length l. The average temperature of the water is T °C and the flow rate is Q cm^3/s. Explain how you would determine the viscous pressure drop along the tube, assuming the water does not change elevation.
33. * BIO **Blood flow through capillaries** Your heart pumps blood at a flow rate of about 80 cm^3/s. The blood flows through approximately 9×10^9 capillaries, each of radius 4×10^{-4} cm and 0.1 cm long. Determine the viscous friction pressure drop across a capillary, assuming a blood viscosity of 4×10^{-3} N·s/m^2.
34. * Determine the ratio of the flow rate through capillary tubes A and B (that is, Q_A/Q_B). The length of A is twice that of B, and the radius of A is one-half that of B. The pressure across both tubes is the same.
35. * A piston pushes 20 °C water through a horizontal tube of 0.20-cm radius and 3.0-m length. One end of the tube is open and at atmospheric pressure. (a) Determine the force needed to push the piston so that the flow rate is 100 cm^3/s. (b) Repeat the problem using SAE 10 oil instead of water.
36. * How can you use the venturi meter system (see Problem 14.28) to determine whether viscous fluid needs an additional pressure difference to flow at the same speed as a nonviscous fluid?
37. * A syringe is filled with water and fixed at the edge of a table at height $h = 1.0$ m above the floor (Figure P14.37). The diameter of the piston is $d_1 = 20.0$ mm, the needle has length $L = 50.00$ mm, and the inner diameter of the needle is $d_2 = 1.0$ mm. You press on the piston so that it moves at constant speed of 10.00 mm/s. Determine (a) the distance D at which the water jet hits the floor and (b) the pressure difference between the ends of the needle. Assume the viscosity of water is 1.0×10^{-3} N·s/m^2.

FIGURE P14.37



14.7 Drag force

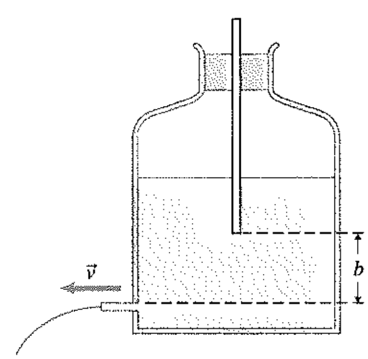
38. **Car drag** A 2300-kg car has a drag coefficient of 0.60 and an effective frontal area of 2.8 m^2. Determine the air drag force on the car when traveling at (a) 24 m/s (55 mi/h) and (b) 31 m/s (70 mi/h).
39. * EST **Air drag when biking** Estimate the drag force opposing your motion when you ride a bicycle at 8 m/s.
40. BIO **Drag on red blood cell** Determine the drag force on an object the size of a red blood cell with a radius of 1.0×10^{-5} m that is moving through 20 °C water at speed 1.0×10^{-5} m/s. (Assume laminar flow.)
41. * BIO EST **Protein terminal speed** A protein of radius 3.0×10^{-9} m falls through a tube of water with viscosity $\eta = 1.0 \times 10^{-3}$ N·s/m^2. Earth exerts a constant downward 3.0×10^{-22}-N force on the protein. (a) Use Stokes's law and the information provided to estimate the terminal speed of the protein. Assume no buoyant force is exerted on the protein. (b) How many hours would be required for the protein to fall 0.10 m?
42. * EST Earth exerts a constant downward force of 7.5×10^{-13} N on a clay particle falling in water. The particle settles 0.10 m in 130 min. Estimate the radius of a clay particle. Assume no buoyant force is exerted on the clay particle. The viscosity of water is 1.0×10^{-3} N·s/m^2.
43. * A sphere falls through a fluid. Earth exerts a constant downward 0.50-N force on the sphere. The fluid exerts an opposing drag force on the fluid given by $F_D = 2v$, where F_D is in newtons if v is in meters per second. Determine the terminal speed of the sphere.
44. * **Terminal speed of balloon** A balloon of mass m drifts down through the air. The air exerts a resistive drag force on the balloon described by the equation $F_D = 0.03v^2$ where F_D is in newtons if v is in meters per second. What is the terminal speed of the balloon?
45. You observe four different liquids (listed with their viscosities and densities in the table below) as they flow with the same speed through identical tubes. You gradually increase the speed of the liquids until their flows become turbulent. Rank the liquids in the order in which you see their flow become turbulent.

Liquid	Viscosity (N·s/m^2)	Density (kg/m^3)
Blood	4×10^{-3}	1.05×10^3
Water	1×10^{-3}	1.00×10^3
Ethanol	1×10^{-3}	0.79×10^3
Acetone	3×10^{-4}	0.79×10^3

General Problems

46. ** Describe how you will design an experimental procedure that will help you decide whether the drag force exerted on a coffee filter falling at terminal velocity depends on the filter's speed v or on its speed squared v^2.
47. * If the speed of air that is blowing across the upper end of a vertical tube is great enough, a thin plate that is placed at the lower end of the tube will remain touching the end of the tube, without falling down. Derive an expression for the minimum speed of air that needs to blow across a tube of diameter d to keep the plate of mass m from falling down.
48. **Mariotte's bottle** Figure P14.48 shows a device called Mariotte's bottle that can deliver a constant flow rate. It consists of a tube that releases air bubbles into a sealed bottle or tank at a height b above the point where the liquid exits. (a) Under what conditions does Mariotte's bottle deliver a constant flow rate and thus a constant speed of liquid? Explain. (b) How does this constant speed of liquid leaving the bottle depend on b, assuming that there is no friction at the opening where liquid leaves the bottle? (Hint: Note that the pressure at the point where the bubbles are entering the bottle is equal to atmospheric pressure.)

FIGURE P14.48



49. ** BIO **Pressure needed for intravenous needle** A glucose solution of viscosity $2.2 \times 10^{-3}\ \text{N} \cdot \text{s/m}^2$ and density $1030\ \text{kg/m}^3$ flows from an elevated open bag into a vein. The needle into the vein has a radius of 0.20 mm and is 3.0 cm long. All other tubes leading to the needle have much larger radii, and viscous forces in them can be ignored. The pressure in the vein is $1000\ \text{N/m}^2$ above atmospheric pressure. (a) Determine the pressure relative to atmospheric pressure needed at the entrance of the needle to maintain a flow rate of $0.10\ \text{cm}^3/\text{s}$. (b) To what elevation should the bag containing the glucose be raised to maintain this pressure at the needle?

50. ** **Viscous friction with Bernoulli** We can include the effect of viscous friction in Bernoulli's equation by adding a term for the thermal energy generated by the viscous retarding force exerted on the fluid. Show that the term to be added to Eq. (14.5) for flow in a vessel of uniform cross-sectional area A is

$$\frac{\Delta U_{\text{Th}}}{V} = \frac{4\pi\eta l v}{A}$$

where v is the average speed of the fluid of viscosity η along the center of a pipe whose length is l.

51. ** (a) Show that the work W done per unit time Δt by viscous friction in a fluid with a flow rate Q across which there is a pressure drop ΔP is

$$\frac{W}{\Delta t} = \Delta P Q = Q^2 R = \frac{\Delta P^2}{R}$$

where $R = 8\eta l/\pi r^4$ is called the *flow resistance* of the fluid moving through a vessel of radius r. (b) By what percentage must the work per unit time increase if the radius of a vessel decreases by 10% and all other quantities including the flow rate remain constant (the pressure does not remain constant)?

52. ** BIO EST **Thermal energy in body due to viscous friction** Estimate the thermal energy generated per second in a normal body due to the viscous friction force in blood as it moves through the circulatory system.

53. ** BIO **Essential hypertension** Suppose your uncle has hypertension that causes the radii of his 40,000,000 arterioles to decrease by 20%. Each arteriole initially was 0.010 mm in radius and 1.0 cm long. By what factor does the resistance $R = 8\eta l/\pi r^4$ to blood flow through an arteriole change because of these decreased radii? The pressure drop across all of the arterioles is about 60 mm Hg. If the flow rate remains the same, what now is the pressure drop change across the arteriole part of the circulatory system?

54. * **Parachutist** A parachutist weighing 80 kg, including the parachute, falls with the parachute open at a constant 8.5-m/s speed toward Earth. The drag coefficient $C_D = 0.50$. What is the area of the parachute?

55. A 0.20-m-radius balloon falls at terminal speed 40 m/s. If the drag coefficient is 0.50, what is the mass of the balloon?

56. ** **Terminal speed of skier** A skier going down a slope of angle θ below the horizontal is opposed by a turbulent drag force that the air exerts on the skier and by a kinetic friction force that the snow exerts on the skier. Show that the terminal speed is

$$v_T = \left[\frac{2mg(\sin\theta - \mu\cos\theta)}{C_D \rho A}\right]^{1/2}$$

where μ is the coefficient of kinetic friction between the skis and the snow, ρ is the density of air, A is the skier's frontal area, and C_D is the drag coefficient.

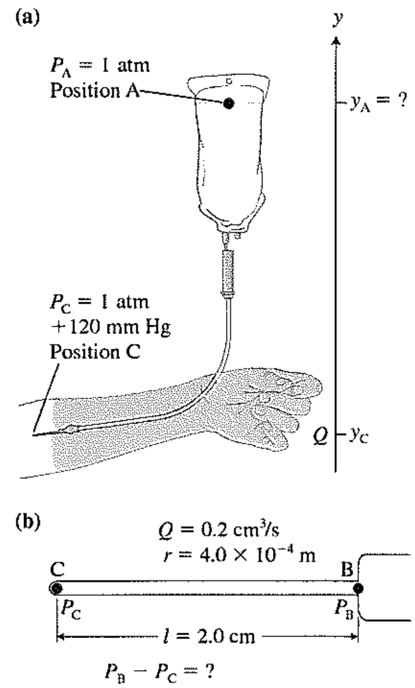
57. ** A grain of sand of radius 0.15 mm and density $2300\ \text{kg/m}^3$ is placed in a 20 °C lake. Determine the terminal speed of the sand as it sinks into the lake. Do not forget to include the buoyant force that the water exerts on the grain.

58. ** EST **Comet crash** On June 30, 1908, a monstrous comet fragment of mass greater than 10^9 kg is thought to have devastated a 2000-km^2 area of remote Siberia (this impact was called the Tunguska event). Estimate the terminal speed of such a comet in air of density $0.70\ \text{kg/m}^3$. State all of your assumptions.

Reading Passage Problems

BIO **Intravenous (IV) feeding** A patient in the hospital needs fluid from a glucose nutrient bag. The glucose solution travels from the bag down a tube and then through a needle inserted into a vein in the patient's arm (Figure 14.12a). Your study of fluid dynamics makes you think that the bag seems a little low above the arm and the narrow needle seems long. You wonder if the glucose is actually making it into the patient's arm. What height should the bag (open at the top) be above the arm so that the glucose solution (density $1000\ \text{kg/m}^3$ and viscosity $1.0 \times 10^{-3}\ \text{N} \cdot \text{s/m}^2$) drains from the open bag down the 0.6-m-long, 2.0×10^{-3}-m radius tube and then through the 0.020-m-long, 4.0×10^{-4}-m radius needle and into the vein? The gauge pressure in the vein in the arm is $+930\ \text{N/m}^2$ (or 7 mm Hg). The nurse says the flow rate should be $0.20 \times 10^{-6}\ \text{m}^3/\text{s}$ ($0.2\ \text{cm}^3/\text{s}$).

FIGURE 14.12 (a) A glucose solution flowing from an open container into a vein. (b) The analysis of the needle in this system.



59. Which answer below is closest to the speed with which the glucose should flow out of the end of the needle at position C in Figure 14.12b?
(a) 0.0004 m/s (b) 0.004 m/s (c) 0.04 m/s (d) 0.4 m/s (e) 4 m/s

60. Which answer below is closest to the speed with which the glucose should flow through the end of the tube just to the right of position B in Figure 14.12b?
(a) 0.0002 m/s (b) 0.002 m/s (c) 0.02 m/s (d) 0.2 m/s (e) 2 m/s

61. Assume that there is no resistive friction pressure drop across the needle (as could be determined using Poiseuille's law). Use the Bernoulli equation and the results from Problems 14.59 and 14.60 to determine which answer below is closest to the change in pressure between positions B and C ($P_B - P_C$) in Figure 14.12b.
(a) $8\ \text{N/m}^2$ (b) $80\ \text{N/m}^2$ (c) $800\ \text{N/m}^2$ (d) $8000\ \text{N/m}^2$ (e) $80{,}000\ \text{N/m}^2$

62. Now, in addition to the Bernoulli pressure change from position B to position C calculated in Problem 14.61, there may be a Poiseuille resistive friction pressure change across the needle from position B to position C. Which answer below is closest to that pressure change?
(a) $0.4\ \text{N/m}^2$ (b) $4\ \text{N/m}^2$ (c) $40\ \text{N/m}^2$ (d) $400\ \text{N/m}^2$ (e) $4000\ \text{N/m}^2$

63. The blood pressure in the vein at position C in Figure 14.12b at the exit of the needle into the blood is $930\ \text{N/m}^2$. Use this value and the results of Problems 14.61 and 14.62 to determine which answer below is closest to the gauge pressure at position B in the tube carrying the glucose to the needle.
(a) $1010\ \text{N/m}^2$ (b) $1410\ \text{N/m}^2$ (c) $1980\ \text{N/m}^2$ (d) $2800\ \text{N/m}^2$ (e) $4620\ \text{N/m}^2$

64. Suppose that there is no Poiseuille resistive friction pressure decrease from the top of the glucose solution in the open bag (position A in Figure 14.12a) through the tube and down to position C near the entrance to the needle. Which answer below is closest to the minimum height of the top of the bag in order for the glucose to flow down from the tube and through the needle into the blood? Remember that the pressure at position A is atmospheric pressure, which is zero gauge pressure.
(a) 0.04 m (b) 0.08 m (c) 0.14 m
(d) 0.27 m (e) 0.60 m
65. Suppose there is a Poiseuille resistive friction pressure decrease from the top of the glucose solution (position A in Figure 14.12a) through the tube and down to position C near the entrance to the needle. How will this affect the placement of the bag relative to the arm?
(a) The bag will need to be higher.
(b) The bag can remain the same height above the arm.
(c) The bag can be placed lower relative to the arm.
(d) Too little information is provided to answer the question.

BIO **The human circulatory system** In the human circulatory system, depicted in Figure 14.13, the heart's left ventricle pumps about 80 cm^3 of blood into the aorta every second. The blood then moves into a larger and larger number of smaller radius vessels (aorta, arteries, arterioles, and capillaries). After the capillaries, which deliver nutrients to the body cells and absorb waste products, the vessels begin to combine into a smaller number of larger radius vessels (venules, small veins, large veins, and finally the vena cava). The vena cava returns blood to the heart (see Table 14.5).

FIGURE 14.13 A schematic representation of the circulatory system including the pressure variation across different types of vessels.

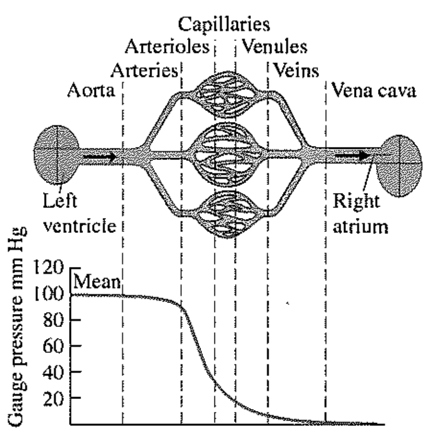


TABLE 14.5 The different types of vessels in the circulatory system

Vessel type	Number of vessels	Approximate radius (mm)
Aorta	1	5
Large arteries	40	2
Smaller arteries	2400	0.4
Arterioles	40,000,000	0.01
Capillaries	1,200,000,000	0.004
Venules	80,000,000	0.02
Small veins	2400	1
Large veins	40	3
Vena cava	1	6

The flow rate Q of blood through the arteries equals the flow rate through the arterioles, which equals the flow rate through the capillaries, and so forth. The average blood gauge pressure in the aorta is about 100 mm Hg. The pressure drops as blood passes through the different groups of vessels and is approximately 0 mm Hg when it returns to the heart at the vena cava.

A working definition of the resistance R to flow by a group of vessels is the ratio of the gauge pressure drop ΔP across those vessels divided by the flow rate Q through the vessels:

$$R = \frac{\Delta P}{Q} \tag{14.12}$$

The gauge pressure drop across the whole system is (100 mm Hg − 0), and the total resistance is

$$R_{\text{total}} = \frac{\Delta P_{\text{total}}}{Q} = \frac{100 \text{ mm Hg}}{80 \text{ cm}^3/\text{s}} = 1.25 \frac{\text{mm Hg}}{\text{cm}^3/\text{s}}$$

The gauge pressure drop across the whole system is the sum of the drops across each type of vessel:

$$\Delta P_{\text{total}} = \Delta P_{\text{aorta}} + \Delta P_{\text{arteries}} + \Delta P_{\text{arterioles}} + \Delta P_{\text{capillaries}} + \cdots + \Delta P_{\text{vena cava}}$$

Now rearrange and insert Eq. (14.12) into the above for the pressure drop across each part:

$$QR_{\text{total}} = QR_{\text{aorta}} + QR_{\text{arteries}} + QR_{\text{arterioles}} + QR_{\text{capillaries}} + \cdots + QR_{\text{vena cava}}$$

Canceling the common flow rate through each group of vessels, we have an expression for the total resistance of the circulatory system:

$$R_{\text{total}} = R_{\text{aorta}} + R_{\text{arteries}} + R_{\text{arterioles}} + R_{\text{capillaries}} + \cdots + R_{\text{vena cava}}$$

The measured gauge pressure drop across the arterioles is about 50 mm Hg, and the arteriole resistance is

$$R_{\text{arterioles}} = \frac{\Delta P_{\text{arterioles}}}{Q} = \frac{50 \text{ mm Hg}}{80 \text{ cm}^3/\text{s}} = 0.62 \frac{\text{mm Hg}}{\text{cm}^3/\text{s}}$$

or about 50% of the total resistance. The next most resistive group of vessels is the capillaries, at about 25% of the total resistance. These percentages vary significantly from person to person. A person with essential hypertension has arterioles and capillaries that are reduced in radius. The resistance to blood flow increases dramatically (the resistance has a $1/r^4$ dependence). The blood pressure has to be greater (for example, double the normal value) in order to produce reasonable flow to the body cells. Even with the increased pressure, the flow rate may still be lower than normal.

66. The capillaries typically produce about 25% of the resistance to blood flow. Which pressure drop below is closest to the pressure drop across the group of capillaries?
(a) 5 mm Hg (b) 15 mm Hg (c) 25 mm Hg
(d) 35 mm Hg (e) 45 mm Hg
67. We found that the arteriole resistance to fluid flow was about 0.62 mm Hg/(cm^3/s). By what factor would you expect the resistance of all the arterioles to change if the radius of each arteriole decreased to 80% of the original value?
(a) 1.3 (b) 1.6 (c) 2.4
(d) 0.4 (e) 0.6
68. Why is the resistance to fluid flow through unobstructed arteries relatively small compared to resistance to fluid flow through the arterioles and capillaries?
(a) The arteries are nearer the heart.
(b) There is a relatively small number of arteries.
(c) The artery radii are relatively large.
(d) b and c
(e) a, b, and c
69. The huge number of capillaries and venules is needed to
(a) provide nutrients (such as O_2) and remove waste products from all of the body cells.
(b) distribute water uniformly throughout the body.
(c) reduce the resistance of the circulatory system.
(d) b and c
(e) a, b, and c
70. Which number below best represents the ratio of the resistance of a single capillary to the resistance of a single arteriole, assuming they are equally long?
(a) 40 (b) 6 (c) 2.5
(d) 0.4 (e) 0.026